Spatial Computable General Equilibrium Modeling
- Static and Dynamic Approaches

Marcus Sundberg

October 2005
Div. of Transport and Location Analysis,
Dept. of Transport and Economics,
Royal Institute of Technology,
SE-100 44 Stockholm, Sweden,
Akademisk avhandling som med tillstånd av Kungliga Tekniska Högskolan framlägges till offentlig granskning för avläggande av teknologe licentiatexamen fredagen den 11 november 2005 kl 10:00, seminarierummet Teknikringen 78 B, KTH, Stockholm.
Abstract

This thesis concerns both static and dynamic modeling in a spatial computable general equilibrium setting. First, we have applied a static framework for the assessment of economic impacts of the Öresund bridge. Secondly, we make an attempt to enhance the static framework through the introduction of economic dynamics.

In the first paper we study the economic impacts of the Öresund bridge. We aim to quantify regional welfare effects as well as effects on regional production and trade. We calibrate a static spatial computable general equilibrium model to economic data representing the Öresund region. In particular we have calibrated a pre-bridge barrier parameter which enables us to study possible barrier reduction effects from the bridge. We present results both as effects of cross strait transport cost reductions and of barrier reductions. It is found that the potential impacts of removing barriers to trade may outweigh the impacts solely due to reduced transport costs.

In the second paper we consider different specifications of a dynamic spatial computable general equilibrium model. In particular we are interested in the effects of different, commonly used assumptions on transition dynamics of such a model. We have specified models with different assumptions regarding capital mobility, the utility specification of the households and assumptions regarding perfect foresight or myopia. A number of simulations have been performed with these models in order to observe policy responses and in order to be able to make specification comparisons of the different models based on those responses. We consider the time it takes for the dynamic models to converge toward the long term steady state to be of importance for welfare assessment of policies and find that convergence may be slow depending on both the model and the policy considered.
Acknowledgments

I would like to express my gratitude to my supervisor Lars Lundqvist. Thank you for your encouragement, great patience, support and for providing sensible advise and comments during the completion of this thesis.

I am grateful to my assistant supervisor Anders Karlström, whom have been a source of true inspiration.

To my family and to Sara, thank you for supporting me on the personal level. You have always been able to guide me and helped me to keep a much needed distance toward my work.

Thanks to all nice colleagues at the division of Transport and Location Analysis. Special thanks to Daniel Jonsson, for fun discussions and for taking time to listen to crazy ideas, especially during the completion of the second paper in this thesis, and to Lars-Göran Mattsson for your sensible leadership.

This research was financed by the Communications Research Board KFB/VINNOVA, the Swedish National Road Administration (Vägverket) and the Swedish National Rail Administration (Banverket), which is gratefully acknowledged.
Contents

1 Why use SCGE models? 1
2 Models in the literature 2
3 Validation 7
4 Scope of this thesis 9
5 Future research 11

List of papers:

(I) Sundberg M, Economic effects of the Öresund bridge - a spatial computable general equilibrium analysis.

(II) Sundberg M, Dynamic spatial CGE frameworks - specifications and simulations.
1 Why use SCGE models?

There has been a large increase of the interest in spatial computable general equilibrium (SCGE) models. One of the reasons for this increase is the demand for tools that may assist in the assessment of policies. In particular for the assessment of the economic impacts of infrastructure investments and policies. One question to be answered is what are the economic impacts of changing accessibility? SCGE models are specifically designed for and have been used for this type of policy assessment. Two reports that indicate this demand for SCGE models are the feasibility study of SCGE models commissioned by the Swedish Institute for Transport and Communications Analysis, and a report commissioned by the UK Department for Transport [10]. The EU has also proven its interest in the SCGE approach by funding the IASON\(^1\) project, recently completed, for documentation see e.g. [2] and [3].

There are numerous ways of approaching the assessment of infrastructure policies. Different approaches such as cost benefit analysis, multi-regional input-output analysis and econometric analysis are all used in assessment. Multi-regional input-output models are the closest relatives to SCGE models, since they attempt to model the economy. Yet, they are not able to fully capture price and quantity effects as they do not allow for substitution effects in the economy. Lowered transport costs due to infrastructure investments tend to stimulate the actors in the economy to take advantage of the cost reduction in production and consumption. These types of effects are difficult to capture within the input-output setting. The strength of input-output models is that they allow a high degree of disaggregation, representing many different regions and types of firms.

If we think of the demand for transports as a derived demand, transport is not necessarily a good in itself but may be a byproduct from the need to move people and goods in space. To treat transports as a derived demand calls for an economy from which to derive it, where essentially the differences in location of supply and demand for goods and services incur transports.

One of the strengths of SCGE models is the use of underpinning microeconomic theory for modeling the economy. The SCGE framework represents a theoretically and mathematically consistent approach toward describing the economy and it allows us to perform policy assessments and forecasts within this consistent framework where the interaction between the economy and transports can be studied.

\(^{1}\)IASON - Integrated Appraisal of Spatial economic and Network effects of transport investments and policies.
2 Models in the literature

The models presented in this section are models that have been implemented and used in policy assessment. The first three models: Pingo, CGEurope and RAEM are examples of static SCGE models with an emphasis on the interplay between transports and economics. The MONASH model and the Diao & Somwaru model are dynamic multi-regional models where the economic actors in the model optimize intertemporal decisions. Following the presentation of these models we discuss different ways of modeling transports and the role of time.

Pingo

The Norwegian model Pingo [12] was developed with the aim of providing forecasts for regional and interregional goods transports. The model uses the assumption of a small open economy with Norway being represented by 19 regions and one rest of the world region, allowing exports and imports. Trade between regions are supported by an explicitly modeled transport sector incurring transport costs. Origin-destination (OD) matrices combined with transport costs from the Norwegian transport model NEMO have been used in order to construct a social accounting matrix, used for calibration of the model. The model distinguishes between 9 types of production sectors, one service sector and one investment sector where the sectors act according to the assumption of perfect competition. Through interaction with the transport model NEMO, forecasts are made for mode specific OD matrices, transport costs, transport volumes etc. Pingo is a static type model, where forecasts are driven by changes in exogenous variables such as policy variables and projections of regional populations.

CGEurope

Earlier versions of the CGEurope model were extended within the IASON project funded by the Fifth Framework RTD Programme, see [3]. This model suite represents one of the largest developments in terms of the number of regions covered, where the whole world is represented but where most of the regions represent parts of Europe, with a total of about one thousand regions. Firms producing tradable goods are modeled in the Dixit-Stiglitz fashion of monopolistic competition [8], while local goods are assumed to be produced under perfect competition. One of the distinctive features of the latest version of CGEurope is that households demand private passenger travel in accordance with utility maximization. Furthermore, freight costs for
goods, international trade barrier costs, and business travel costs are included in the transaction costs of the firms. Similar to the idea in Pingo, transport costs are derived from the separate transport model SCENES, which is a model providing forecasts of freight and personal travel costs. The principal aim of the CGEurope model is to provide spatially distributed welfare effects linked to changes in accessibility within and between regions.

**RAEM**

The Dutch model, RAEM [21], is a model or framework under development. After some preliminary experience with early versions of the model a number of issues with regard to model assumptions were raised, and ideas of how to resolve these issues have led to model development. In [20] these issues are put forward as "1) interfacing problems between transport and SCGE models, 2) the modeling of the influence of transport costs on sectoral production, 3) the interpretation of the conventional, micro-level specification of product variety in aggregate applications and 4) the problem of irrational agglomeration effects in economic activities". The first issue refers to problems with different definitions of transport costs in SCGE and transport models and problems with regard to the transport accounts of firms that are typically not represented in the data to which SCGE models are calibrated. The second issue is a critique of the commonly used iceberg approach [19] to model transport costs. It is argued that this approach may suffice in aggregate models but may lead to serious mis-specifications of transport costs in sectorally disaggregated models. The last two problems arise from assumptions of monopolistic competition in the Dixit-Stiglitz fashion [8]. Using this approach enables the study of agglomeration effects, but there are issues to address regarding problems of 'irrational agglomeration effects'. These aspects are not fundamental problems to the approach taken in the development of RAEM, but rather constructive critique of shortcomings with the approach, where possible ways of dealing with these issues are reported in [20]. In RAEM, commuting between regions is implicitly modeled through applying Pissarides search model, e.g. see [15], in describing the labor market, where search for jobs in other regions than the region of residence creates a commuting pattern.

**MONASH**

The MONASH model represents a long tradition of CGE modeling at the Center of Policy Studies and Impact Project at Monash University, Australia. The model is an extension of the earlier developed static ORANI model which
MODELS IN THE LITERATURE

MONASH is a multi-regional, multi-sectoral dynamic CGE model system which allows for different choices of the levels of sectoral and regional disaggregation, for an overview see [7]. Also, the model can be applied with different assumptions regarding the dynamic behavior of economic agents, including the assumptions of perfect foresight or static expectations. The transport sectors are identified as margin sectors which are required for trade of goods and services in the model, where the costs imposed by the margin sectors are included in the purchase price of tradables. Both firms and households are assumed to make intertemporal decisions, where firms maximize their values by investments in their capital stocks and acts in a perfectly competitive environment and households make consumption versus savings decisions. Another feature of the MONASH system is that it allows for different types of closures. This means that depending on the situation where the model is used, different assumptions regarding which variables that are exogenously given and which are endogenously determined within the model can be adopted.

Dixon and Rimmer [7] states the ideas that: "(a) CGE models can be used in forecasting; and (b) forecasts matter for policy analysis", and they note that assessment of possible policies requires realistic base case scenarios. This need for realistic forecasts is supported by the fact that in the dynamic setting the order and timing of different exogenous changes and policies matter for the transition path of the economy.

Diao & Somwaru

The Diao & Somwaru model [6] was constructed to analyze effects of the MERCOSUR (Southern Common Market including four South American countries). This model is a multi-regional, multi-sectoral, dynamic CGE model which is an extension and application of the model presented in [5]. In principle these models adopt the same assumptions of intertemporal optimization by firms and households as is used in the MONASH model. Yet, the descriptions of these models are more accessible and we have found them inspirational for developing the spatial dynamic model tested in this thesis. Even though the Diao & Somwaru model is multi-regional it is not spatial, since there are no transport costs inferred on trade, in contrast to all the previously described models.

Modeling of transport costs

In all the models described above, except the Diao & Somwaru model, transport costs are included in actions of trade. In CGEurope the costs of trans-
ports are represented by adopting Samuelson’s iceberg approach [19], a way of modeling transport costs without an explicit representation of a transport sector. In Pingo, RAEM and MONASH the transport sectors are more or less explicitly modeled, and the transport costs are added to the price of purchasing goods and services from different regions. In the cases of Pingo, CGEurope and RAEM the basic approach is to make use of a transport network model external to the SCGE model in providing these transport costs. As shown in Figure 1 in a simplified manner, a scenario or policy may be implemented and the resulting effects in the transport network model on e.g. transport costs may be fed into the SCGE model which may produce results to be fed back to the transport model. The idea is to do this in an iterative manner until an equilibrium between the two model systems is achieved.

An alternative approach was described theoretically in [9], where a freight network model and a SCGE model were combined in a simultaneous model. This approach was implemented in a test model in [13], where network models for car transport and public transports as well as a freight network model were merged with a SCGE model.

Figure 1: Typical interaction patterns between a transport network model and a SCGE model in policy assessment.
What about time?

In the static frameworks temporal aspects are most commonly not addressed. The usual procedure in the application of a static models is to calibrate the model to data for a benchmark year yielding a benchmark equilibrium of the model. Then, policy analysis is performed by applying the policy in the model and compute a counterfactual equilibrium. The results from the two static equilibria are compared to reveal the implications of the policy.

One natural candidate to the ceteris paribus, all else being equal, approach is to construct a basecase scenario with a corresponding basecase forecast equilibrium to be compared to a forecast equilibrium with the intended policy included. In a dynamic setting, not only the basecase scenario would matter, but the temporal ordering and timing of different policies make a difference for economic transitions. In principle, if we think of two hypothetical 'policies' A and B implemented in sequence, where policy A is to keep households’ welfare constant over ten years and policy B is to increase the households’ welfare by ten percent over ten years. Then, households would be better off having policy B implemented before policy A than vice versa, as they achieve the higher level of welfare during the implementation of policy A. This highly simplified example illustrates the importance of ordering of changes or policies in a temporal economy.

Another aspect of time in economic modeling is the time it takes for the modeled economy to adjust toward an equilibrium, a steady state. In static models this question is dealt with by adjusting the assumptions made in the model to fit the intended time range wherein the model is applied. For example, in a short term model the assumption of immobility of the labor force between regions may be adopted, whereas in a long term model the labor force may be assumed mobile between regions. One question is then, how shall we interpret short and long term? Usually this question is left to the modelers intuition and judgment. Without making any statements for the real economy, when simulating dynamic models we can test how transition times toward equilibrium are affected by e.g. mobility assumptions. Then it is primarily an empiric question which assumptions we should adopt and which model fits real world data the best.
3 Validation

For transport model systems Lundqvist and Mattsson [16] divide model validation into four parts:

- **Practical validation.** What is the system level? Which mechanisms are exogenous and which are endogenous?

- **Theoretical validation.** What is the theoretical foundation? Does the system use an equilibrium or dynamic approach? Are the various causal relationships reasonably well modeled?

- **Internal validation.** How good are the models at reproducing the data on which they have been estimated? Is the responsiveness to changes in explanatory variables reasonable?

- **External validation.** Can the model system reasonably well reproduce other independent data? How well can the model system reproduce a future year (forecasting) or a previous year (backcasting)?

In terms of these classifications, the strength of computable general equilibrium models pertain to particularly theoretic validation. One of the main aims of CGE models is to rely on economic theory to model causal relationships in the economy. In doing this most models represent the economic actors in an aggregate fashion. For representing heterogeneous households in an aggregate fashion preferences must satisfy the Gorman form, see [22].

There are similar results for representing firms in an aggregate fashion and with regard to these results on aggregation Hansen and Heckman [11] note: "These results give examples of when simple aggregate relations can be deduced from relations underlying the micro behavior of the individual agents, but they do not justify using the constructed aggregate relations to evaluate fully the welfare costs and benefits of a policy". Ideally the results from an aggregate model should be disaggregated back to the individual agents. This task is quite demanding and in practice we may have to rely on aggregate results.

Internal and external validations are scarcely employed, at best, within the field of CGE modeling. Most models, to our knowledge, are calibrated to data in such a way that the parameters of the models are just identified. That is, calibration makes use of as many observations of data as there are parameters in the model. Hence there is usually little knowledge of the uncertainties in calibrated parameters. Due to this uncertainty a common approach is to perform sensitivity analysis with respect to the calibrated parameters, yielding the sensitivity of model results to variations in parameters,
see [4] for an example. McKitrick [17] performs a type of internal validation of CGE models when he tests the role of functional forms in the models. He finds that the model results are sensitive to the functional forms chosen to represent the agents in the economy, this result being one of his platforms for questioning the empirical relevance of CGE models. This type of criticism highlights issues that need to be addressed within the field of CGE modeling. One example of external validation can be found in the work by Kehoe [14]. He performs ex post evaluations of a number of CGE forecasts, made with different models assessing the economic impacts of NAFTA (the North American Free Trade Agreement). More studies of this sort are called for as they may provide information on model weaknesses and directions for model improvements.
4 Scope of this thesis

In paper I we apply a static spatial computable general equilibrium model to assess the economic impacts of the Öresund bridge. In this application we have primarily addressed two issues in the development of the model, the first being how to construct a useful data set for calibrating the model, and the second issue being how to implement a cross strait trade barrier in the model.

Concerning the construction of data, different approaches could be considered. Either a regionalized social accounting matrix, describing economic activities for all regions in a disaggregate fashion, or an input-output matrix describing the economy on a regionally aggregated level could be used for calibration. The former approach requires trade data for both inter-and intra-regional trade, data that were not available in the Öresund case. The latter requires data for the modeled economy as a whole, or regionally aggregated, and rests on assumptions of equal preferences of households and equal technologies of firms across regions, neither was this type of data available for the Öresund region. Therefore we developed a methodology to construct the needed data using matrix balancing methods. In doing this we were able to set up two sets of data allowing us to identify differences in preferences and technologies between the Danish and Swedish side of the Öresund region, retaining the assumption of equal preferences and technologies across regions within the respective countries.

When introducing the trade barrier between Sweden and Denmark to the Öresund model we need to identify the meaning of this barrier. Some possible foundations for barriers to trade within the Öresund model are:

- Distance dependent trade costs that are not accounted for.
- Differences in legislation, language and culture.
- Other trade related costs.

The model uses euclidean distances between regions to represent the distance related costs of trade. This means that the extra cost of crossing the Öresund strait is not accounted for in the distance related trade cost of the model. Since the extra cost of relying on ferries, for cross strait trade, is not accounted for in the distance dependent costs of the model, these costs may be captured by the modeled barrier. Differences in legislation, language and culture are examples of barriers that may incur frictions to trade. Finally, other trade related costs, not represented in the model, are costs due to logistics, personal travel and business travel. All these different factors
may be accounted for by the introduction of the trade barrier in the model. The modeled barrier was consistently calibrated within the SCGE framework such that cross strait trade coincides with an observation of the cross strait trade pattern on an aggregate level. In paper I we provide resulting effects of reducing this trade barrier. It is clear that these results need to be interpreted with some caution, as we are not able to identify different parts of barrier costs. We have no means of distinguishing between the trade costs that are not accounted for and parts of the trade barrier due to differences in language for example. Hence the results concerning removal of the trade barrier can be viewed partly as a sensitivity analysis of the model to the trade barrier and partly as describing the potential economic impacts of removing such a barrier.

In paper II we make a first attempt to address issues of time within a SCGE framework. As discussed earlier temporal issues may be important both for forecasting and policy analysis. Our approach has been that ‘small is beautiful’. Therefore we have stated a small scale test model where we merge a static SCGE framework [1] with a classical framework for intertemporal consumption/savings modeling [18]. Previous static type SCGE models typically take the capital stocks used for production as exogenous to the model. In doing this they implicitly assume away the possibility to study investment effects, and welfare effects thereof, due to infrastructure improvements for example. Within the developed framework this is one possible issue that can be addressed, the formation of capital due to different policies.

Apart from understanding how to model the temporal economy and stating the model, we aim to test the behavior of the model with regard to different but commonly used assumptions. We have focused on studying the dynamic behavior of the economy with respect to different assumptions on capital mobility and consumer behavior. Consumers are modeled both as agents acting according to perfect foresight and static expectations. In the perfect foresight specification we can separate the time of announcement and the time of implementation of policies allowing the households to react to the implications of future policies. In the static expectations specification this type of anticipation can not be studied as the households only react to a policy when it has been implemented. An alternative to perfect foresight or static expectations is extrapolative expectations. This represents a way of letting expectations of the future be derived from the history of the economy, a specification we have not yet tested. As the model tested is a small scale first attempt toward stating a dynamic SCGE it is clear that the model can be extended.
5 Future research

Our objective for the future is to implement a dynamic SCGE for Sweden, to be used in forecasting of freight transport. There are a multitude of modeling issues that may be addressed for this purpose. Some possible directions for future work are:

- Dynamics in a small open economy.
- Explicit representation of transport sector.
- Monopolistic competition.
- Migration and labor markets.
- Calibration and Data.

In the dynamic model presented in this thesis we used the closed economy assumption. Implementing a model for Sweden necessitates using the assumption of a small open economy that is, Swedish regions trade goods and services with other countries as well. If we want to model capital as a regionally mobile production factor this has some implications in an open economy. If the productivity of using capital in Sweden differ from that of using it in other countries we would expect rapid geographical redistributions of the usage of capital, as it would be used where it is most profitable. To avoid this type of violent behavior, in the CGE literature this is usually referred to as bang-bang solutions, we will have to introduce some type of adjustment costs for relocating capital between regions and countries.

In the present thesis we have used a simple fashion of representing the transport sector by adopting Samuelson’s iceberg approach. Due to the valid criticism of this approach presented in [20] it would be preferable to move toward a more explicit representation of the transport sector. The iceberg approach may suffice in aggregate one sector models but is prone to misspecification in a multi-sectoral setting.

We have specified markets to act under perfect competition, another common specification is to assume that markets are monopolistically competitive. In models with monopolistic competition it is possible to study agglomeration effects in the economy. There are a number of agglomeration forces in a monopolistic setting, to summarize [21] with regard to firms: (1) Monopolistic firms try to locate themselves in big markets to (a) minimize transport costs of inputs and (b) to gain in productivity via a larger variety of inputs, in order to be competitive. (2) Monopolistic firms have an incentive to locate themselves in regions with few competitors to avoid competition.
Migration could be made endogenous to the model, in the present specifications of our models regional populations have been exogenous. The same holds for labor supply.

Calibration and data issues always have to be dealt with when implementing a model. The availability of relevant data puts restrictions on what may and what may not be modeled. In the application of the Öresund model we gained knowledge on this interplay between using relevant data and specifying the model. Moving toward estimation of SCGE models is another area which could be addressed.
References


Economic Effects of the Öresund Bridge
- A spatial computable general equilibrium analysis

Marcus Sundberg

October 2005
Div. of Transport and Location Analysis,
Dept. of Transport and Economics,
Royal Institute of Technology,
SE-100 44 Stockholm, Sweden,
Abstract

In this paper we study the economic impacts of the Öresund bridge. We aim to quantify regional welfare effects as well as effects on regional production and trade. We calibrate a static spatial computable general equilibrium model to economic data representing the Öresund region. In particular we have calibrated a pre-bridge barrier parameter which enables us to study possible barrier reduction effects from the bridge. We present results both as effects of cross strait transport cost reductions and of barrier reductions.
Contents

1 Introduction 1

2 The Model 1
   2.1 Description .............................................. 1
   2.2 Firms .................................................. 4
   2.3 Transport agents with barriers to trade ............... 5
   2.4 Households ............................................. 7
   2.5 Rest of the world ....................................... 8
   2.6 Market clearing ......................................... 8

3 Calibration Methodology 9
   3.1 SAM and IO calibration .................................. 9
   3.2 Öresund SCGE calibration ............................... 10
      3.2.1 Data ................................................ 11
      3.2.2 Calibration ........................................ 15

4 Results 17

5 Conclusions 22

A SCGE Trade Barriers 25
   A.1 The reduced SCGE model ............................... 25
   A.2 Results ................................................ 28
   A.3 Conclusions ........................................... 31

B Notation 33

C Data 34
1 Introduction

This paper is a study on the economic effects due to the Öresund bridge, on a regional level. The approach we have chosen for this study is based on the general equilibrium framework of economic theory. This will allow us to develop a closed consistent framework for studying the regarded issues. This framework will allow us to study the regional economic effects due to reduction in cross strait transport costs. Also, other trade barriers than transport costs will be considered, and the economic effects due to trade barrier reductions will be analyzed.

The model stated in this paper has been calibrated to the specific economic circumstances in the Öresund region. Within this model we have been able to consistently calibrate a pre-bridge trade barrier, which allows us to study the previously mentioned barrier effects. It will be shown that barrier reductions may have larger potential than the pure transport cost reductions in economic terms and in welfare terms. This is interesting in the context of the results found by Bröcker [5], also within a CGE (Computable General Equilibrium) framework, which did not concern barrier effects as such, but only effects due to changes in transport costs.

The work presented here is part of a larger study of the effects of the Öresund fixed link. Results from this larger study can be found in [1],[11] and [12]. In these references numerous studies regarding regional economic impacts, barriers to trade, commuting and transports, accessibility and productivity consequences can be found.

2 The Model

In this section we will give a technical description of the spatial computable general equilibrium (SCGE) model used. The basic structure of the model we use was developed by Bröcker [4]. The model has been extended in some aspects: treatment of trade with the rest of the world, possibility of including barriers to trade, and calibration procedures to fit available data. Before going into the technicalities of the model, we give an informal description of building blocks of the model and their relations to each other.

2.1 Description

The Öresund region has been divided into five regions, three on the Swedish side, and two on the Danish side of the Öresund strait. The Swedish regions are Helsingborg (1), Kristianstad (2), and Malmö (3), and the Danish regions
are Copenhagen (4) and West Zealand/Storstrøm (5). This regionalization is depicted in Figure 1. Formally, within the model there is a sixth region representing the rest of the world, which gives us a model of a small open economy. Each region shelters a number of economic actors, creating the economic activity of the region. These actors are households, firms, and transport agents.

The households in one region are represented in an aggregate fashion, through one representative household. The household owns production factors such as labor and capital. By supplying these production factors to firms within the region the representative household earns its income. This income constitutes the household budget which is spent on consumption of the different types of goods and services, offered by the firms. The consumption decision is made in order to maximize utility.

There are a number of firms represented in each region. Every firm is responsible for the production of a single type of output. In order to produce this output it uses primary and intermediate inputs. The primary inputs are the production factors provided by the households, and intermediate inputs are goods and services being purchased from all the different sectors. In the model implemented here we have five different sectors represented: Manufacture I, Manufacture II, Construction, Sales and Repair, and Service. Each of these sectors is represented by a firm in every region. The firms act as profit maximizers under perfect competition.

Corresponding to the set of firms, there are transport agents. A particular
2.1 Description

The transport agent is responsible for transporting one type of sector output, from all regions, to the region of destination. This specification implies that outputs from firms in a specific sector are not seen as perfect substitutes, this is usually referred to as the Armington assumption [2]. Hence, trade in all outputs may occur from all regions to all regions. Transport costs are included according to Samuelsons’ iceberg principle [13], which assumes that part of the commodity transported is used up during the transportation. In short, the task of the transport agent is twofold: transporting output and merging outputs into a composite good. The iceberg principle is used to model distance dependent trade costs, but the model has been adapted to deal with barriers to trade that are not distance dependent. Examples of such barriers would be differences in legislation, culture, language etc.

![Diagram of the SCGE-model](image)

Figure 2: Real flows in the SCGE-model. Financial flows are opposite in direction.

In Figure 2 we have depicted real flows for a specific sector of a region. Finally, all markets in the model must clear. All production factors provided by the households are used by the firms and all output of firms are used either as intermediate inputs by firms or consumed by the households. We now turn to a more formal description of the model.
2.2 Firms

The firm in sector \( j \) in region \( s \) is responsible for producing output of good \( j \) in that region. For its production the firm uses regional pool goods from all sectors as intermediate inputs and factors, owned by households, as primary inputs. The firm’s technology is specified through a NCES\(^1\) unit cost function \( cf^j(q_s, w_s; \alpha^j_s, \gamma^j_s) \), which represents the minimum cost of producing one unit of output at input prices \( q_s := (q^1_s, \ldots, q^I_s) \) and \( w_s := (w^1_s, \ldots, w^K_s) \). The vectors \( \alpha^j_s \) and \( \gamma^j_s \) are the so called position parameters to be calibrated to benchmark data. Note that the parameters of the cost function carry both indices \( j \) and \( s \), that is, firms in sector \( j \) uses different technologies in different regions. We have used the assumption that technologies do not vary within a country. In our model with two ”countries”, the Swedish and the Danish part of the Öresund region, we have two corresponding technologies for each sector.

Due to perfect competition and free entry to the markets, prices equal minimum cost of production:

\[
p^j_s = cf^j(q_s, w_s; \alpha^j_s, \gamma^j_s), \tag{1}
\]

where \( p^j_s \) is the unit price of good \( j \) produced in region \( s \), since \( cf^j(\cdot) \) is the unit cost function. Then according to Shephard’s lemma, the input coefficients are

\[
a^{ij}_s = \frac{\partial cf^j(q_s, w_s; \alpha^j_s, \gamma^j_s)}{\partial q^i_s}, \tag{2}
\]

\[
c^{kj}_s = \frac{\partial cf^j(q_s, w_s; \alpha^j_s, \gamma^j_s)}{\partial w^k_s}. \tag{3}
\]

The input coefficient \( a^{ij}_s \) is the amount needed of pool good \( i \) in order to be able to produce one unit of output of good \( j \), in region \( s \). Similarly, \( c^{kj}_s \) is the amount of factor \( k \) that is needed per unit of good \( j \) produced in region \( s \), where factor \( k \) is a primary input such as labor or capital.

The nesting structure, or substitution structure of the firms cost functions is taken exogenous to the model. In Figure 4 this structure is shown, where we have chosen a Leontief\(^2\) structure between intermediate inputs combined with a CES\(^3\) structure for substitution between primary inputs.

---

\(^1\)NCES stands for Nested Constant Elasticity of Substitution, for a detailed description of this class of functions see [4].

\(^2\)This implies that there is no substitution at this level.

\(^3\)CES stands for Constant Elasticity of Substitution, this is a special case of a NCES.
2.3 Transport agents with barriers to trade

The transport agent of a region is responsible for transforming output of all regions into a composite commodity, a pool good. The pool good of kind $i$ in region $r$ is made up of output of kind $i$ from all regions. Transport costs are included in the model through the iceberg principle, through which goods are partly used up in the process of transportation. Barriers to trade are modeled in a similar fashion, as a price wedge or tariff equivalent, following Deardorff and Stern [8]. A commodity that is transported across a barrier is subject to a non-rent-bearing trade cost i.e. similar to the transport cost it is wasted. One can think of such a cost to represent the obstacle of overcoming differences in language, legislation, culture etc. In our application we model such a barrier to trade between the Swedish and the Danish part of the Öresund region.

The activity of a transport agent is specified through a NCES unit cost-function $c^\tilde{\eta}(v^i_s; \vartheta^i)$, where $v^i_s = \{v^i_{r1}, \ldots, v^i_{rK}\}$ is the vector of prices by sending region $r$ facing the agent in region $s$, and $\vartheta^i = \{\vartheta^i_1, \ldots, \vartheta^i_R\}$ is the corresponding parameter vector. Due to the iceberg principle, and the barrier price wedge, these prices are given by

$$v^i_{rs} = p^i_r, \beta_{rs} \exp(\eta^i z_{rs}),$$

where $v^i_{rs}$ is the price of good of type $i$ from region $r$, perceived by the transport agent located in region $s$. The parameter $\eta^i$ is the commodity specific transport rate and $z_{rs}$ is the distance between region $r$ and $s$. The barrier to trade is incorporated through $\beta_{rs}$ which we define as

$$\beta_{rs} \equiv \begin{cases} 
\beta, & \text{if } r \in S \text{ and } s \in D \text{ or vice versa} \\
1, & \text{otherwise},
\end{cases}$$
where $\beta$ is a barrier parameter to be calibrated. In the presence of a barrier we expect $\beta > 1$. The value $(\beta - 1)$ is what we will refer to as the tariff equivalent, since it describes the percentage increase in prices of cross barrier traded goods. The purpose of the barrier parameter is simply to skew prices in such a way that the transport agents shifts away from goods produced in the other country, lowering the trade between Swedish and Danish regions to the observed level.

Define $ct^i_s(p^i; \vartheta^i) := c\tilde{t}^i_s(v^i_s; \vartheta^i)$, then in equilibrium the price of the pool good equal the price of the composite commodity including transport and barrier costs

$$q^i_s = ct^i_s(p^i; \vartheta^i).$$

The delivery of sector $i$ from region $r$ to $s$ per unit pool-good of region $s$ is

$$t^i_{rs} = \frac{\partial ct^i_s(p^i; \vartheta^i)}{\partial p^i_r}.$$ (5)

Regarding the technologies of transport agent $i$ in region $s$, we assume that there is a common underlying technology $\hat{\vartheta}^i$ for all regions. That is, goods from different regions are perceived the same way in all regions. An other way of thinking about this is that goods are demanded with regard to their origin, the parameter vector $\hat{\vartheta}^i = \{\hat{\vartheta}^i_1, \ldots, \hat{\vartheta}^i_R\}$ concerns only the origin $r \in \{1, \ldots, R\}$ of a good and not its destination $s$. Clearly the actual quantities demanded, for producing one unit of pool good, will vary by both region of origin and destination since the prices $v^i_{rs}$ do vary by origin and destination. The substitution structure of the transport agents is shown in Figure 4.

Figure 4: The transport agents’ substitution structure of NCES-type. Region $R$ refers to a rest of the world region.
2.4 Households

Each region is considered to shelter one representative household. The household earn its income from providing exogenous fixed amounts of factor services $f^k_s$ to firms, and uses this income to create the final demand in the model. Factor services provided by the household are sold to firms within the region of residence. Preferences of the household are completely determined by its expenditure function $eh(q_s, u_s)$ which states the minimum expenditure required to achieve utility level $u_s$ at prices $q_s$. The expenditure function has the form

$$eh(q_s, u_s) = u_s ch(q_s; \delta_s),$$

(6)

where $ch(q_s; \delta)$ is a CES cost function with parameter vector $\delta = (\delta^1_s, \ldots, \delta^I_s)$. Household preferences differ between countries, but within a country preferences are assumed to be the same for all regions.

![CES Diagram](https://via.placeholder.com/150)

Figure 5: The household substitution structure of CES-type.

The household in region $s$ spends all its income on final demand, in equilibrium that means

$$y_s = eh(q_s, u_s),$$

(7)

where the expenditure is evaluated at the pool prices. Furthermore, demand is given by

$$d_s^i = \frac{\partial eh(q_s, u_s)}{\partial q_i^s},$$

(8)

The income earned by the household in region $s$ is given by adding incomes from the different factors of production owned by the household

$$y_s = \sum_k w^k_s f^k_s.$$
2.5 Rest of the world

The rest of the world region, $R$, has different properties from the other regions. It does not hold firms or households as the other regions, but rather it has a separate function in creating export demand and import supply with regard to the other regions. Prices in the rest of the world are taken as exogenous and they are defined to be unity

$$p^i_R \equiv 1.$$  \hspace{1cm} (10)

Import supply from the rest of the world is completely elastic, and is the output $S^i$ provided by the region. Note that the quantities and values of import supply coincides as the price is set to one.

Export demand volume in rest of the world is modeled as

$$E^i = \zeta^i (q^i_R)^{-\varepsilon},$$  \hspace{1cm} (11)

where $\zeta^i$ is a level parameter to be calibrated, and $\varepsilon$ is a given elasticity. The exported volume from region $r$ to $R$ is determined by the transport agent in $R$ as

$$E^i_r = t^i_{rR} E^i_r,$$  \hspace{1cm} (12)

and the export value from $r$ is given as $p^i_r E^i_r$.

2.6 Market clearing

Factor markets clear, all supplied factors from the household in region $s$ are used as primary inputs by firms in the same region

$$f^k_s = \sum_j c^k_{sj} x^j_s,$$  \hspace{1cm} (13)

and outputs are determined by the input/output (I/O) equations

$$x^i_r = \sum_{s=1}^{R-1} t^i_{rs} (d^i_s + \sum_j a^ij_s x^j_s) + E^i_r, \hspace{0.5cm} r = 1..R - 1,$$  \hspace{1cm} (14)

$$S^i = \sum_{s=1}^{R-1} t^i_{Rs} (d^i_s + \sum_j a^ij_s x^j_s).$$  \hspace{1cm} (15)

The output of firm $i$ in region $r$ is either transported to the Öresund regions, where it is used to satisfy final demand by the households and as intermediate inputs to the firms, or it is used to satisfy export demand from rest of the world. The rest of the world region is not allowed to trade with itself, hence the special treatment of that region. Equations (1)-(15) constitute the non-linear system of the model.
3 Calibration Methodology

Ideally, the supply of data is abundant and computational complexity is not a concern. Clearly, we would like to find parameters of our model that best fits data in some sense. We can think of the model presented in Section 2 as a vector valued function $F(x, a)$, where $x$ is the collection of all endogenous variables of the model and $a$ is the collection of all parameters. The equilibrium can be characterized as

$$F(x, a) = 0$$

Let $y$ be some vector of observed data, and $L$ be an estimator. Then, typically we could express the calibration of the model as a maximization problem

$$\max_a L(y, x, a),$$

$$s.t. \quad F(x, a) = 0.$$ 

In this type of calibration, we think of $y$ containing as much relevant data as possible, in particular one would make use of time series of data when possible. The tradition of calibrating applied general equilibrium models usually does not concern this type of exploration of data. It is more common that a minimum data set is used, such that the model parameters can be identified. There are different practices regarding calibration approaches for spatial computable general equilibrium models, two such practices will be described below.

3.1 SAM and IO calibration

One common approach to calibrating a SCGE model is to set up a regionalized social accounting matrix (SAM). The SAM represents economic activities in the economy where typically one column represents the output and inputs of a specific firm or transport agent in a specific region. Rows represent market clearing conditions. In order to achieve consistency between a SAM and an equilibrium model rows and columns sum to zero, this can be seen by the market clearing conditions and zero profit conditions. A typical SAM has the structure shown below in Figure 6.

From the structure of the SAM it is obvious that the need for data on sectoral and regional trade is large. The trade pattern for each region and sector has to be known with regard to all other regions. Two examples of models that have been calibrated to such data sets are the Norwegian model Pingo [10] and the Danish model Brobisse [7].
3.2 Öresund SCGE calibration

In the calibration of the Öresund SCGE model we have taken a compromise approach. As we lack the needed data on interregional trade we are not able to set up a full SAM scheme. Yet, we have data on both the Swedish and Danish side of the Öresund region under study. We make assumptions of common technologies and preferences for regions belonging to the same country, allowing us to calibrate different production technologies and preferences for the different countries. We will set up two separate sets of input systems, one for the Swedish regions and one for the Danish. In the next section we describe the treatment of data to achieve these sets, and in Section 3.2.2 we state the equations used for calibration to the data sets.
3.2.1 Data

The original type calibration of the Bröcker model could be used to calibrate the Öresund model. This requires Input-Output data of the modeled regionalized closed economy

\[
\begin{array}{ccc}
A^{11} & \ldots & A^{1I} & D^1 \\
\vdots & \ddots & \vdots & \vdots \\
A^{I1} & \ldots & A^{II} & D^I \\
C^{11} & \ldots & C^{1l} \\
\vdots & \ddots & \vdots \\
C^{K1} & \ldots & C^{KI}.
\end{array}
\]

In this data set \(A^{ij}\) denotes aggregate sales values from sector \(i\) to sector \(j\), \(C^{kj}\) is the aggregate value of primary factor input of type \(k\) to sector \(j\) and \(D^i\) is the aggregate final demand by the households. Also, data on regional labor input to the sectors, and regional factor prices for each factor input is needed

\[
\bar{w}^k_r \quad \forall \ k, r \\
L^i_r \quad \forall \ i, r.
\]

The idea of obtaining our data set is to make use of two sources of data. The first source is the IO-table used for Scania in RAPS, which reports IO-data on the basis of a 49 sector division, and with two factor inputs in form of labor and capital. The capital part of factor input in the IO-table has been taken as the residual of the row/column-sums i.e. it includes firms’ profits, land use related costs, and all other costs not accounted for in the IO-table. The second source of data is the Örestat database. In this database one can find specific data for the Öresund region. The following data from the base year 1999 are used:

- **Turnover** \((T)\), given for 17 sectors and 2 regions being the Swedish and Danish side of the Öresund region.

- **Value added** \((V)\), also for 17 sectors and the same 2 regions.

- **Employment** \((L)\), which is reported in Örestat either by 17 sectors, or by 33 municipalities in the Swedish part of the Öresund region. For the Danish side, employment is reported for the 17 sectors at a regional level of 7 counties.
The data from the two different sources must be merged into one consistent set of data for the Öresund region. First one notes that the sectors are represented on different levels of aggregation in the two sources, 49 in RAPS and 17 in Örestat. Therefore the RAPS sectors have been aggregated to match the ones in Örestat. In Örestat we can find data on at least the regional level of Scania and Copenhagen. Therefore we are able to separate the IO-system into two systems representing these two regions. The original RAPS IO-system is used as a starting point, representing a priori information on the different sectors’ technologies. This system is then balanced, using the minimum information principle, to match the observed data in Örestat. The resulting two parallel input systems, one for Scania and one for the Copenhagen region, are:

$$
\begin{array}{ccc}
A_S^{11} & \ldots & A_S^{1I} \\
\vdots & \ddots & \vdots \\
A_S^{I1} & \ldots & A_S^{II} \\
\end{array}
\begin{array}{c}
D_S^1 \\
\vdots \\
D_S^I \\
\end{array}
\begin{array}{ccc}
A_D^{11} & \ldots & A_D^{1I} \\
\vdots & \ddots & \vdots \\
A_D^{I1} & \ldots & A_D^{II} \\
\end{array}
\begin{array}{c}
D_D^1 \\
\vdots \\
D_D^I \\
\end{array}

Here the labels $S$ and $D$ are used to indicate if data concerns the Swedish or Danish side of the Öresund region.

The factor input matrix $\hat{C}^{kj}$ originating from RAPS has been balanced or rather scaled, with scaling parameters $\gamma$, such that every column sum adds up to the value added for that sector. It holds that,

$$V_j^o = \gamma_j^o \sum_{k \in K} \hat{C}^{kj}, \quad \forall j$$

where the regional labels have been suppressed. We use the ”knot” notation referring to the two sets ($o = \{D, S\}$). The last expression contains a $\hat{C}$, which is the notation we will use to indicate that something is originating from RAPS data. The scaled matrices are then given by

$$C^{kj}_o = \gamma_j^o \hat{C}^{kj}, \quad \forall k, j$$

Furthermore, it holds for every sector in the IO-table that total inputs equal total outputs, which is the total turnover. The matrix

$$[\hat{A} \hat{D}]$$
from RAPS is rebalanced, with balancing parameters \( \lambda \) and \( \mu \) such that

\[
T_o^i = \lambda_o^i \left( \sum_{j \in I} \mu^j_o \hat{A}^{ij} + \mu^{I+1}_o \hat{D}^i \right) \quad \forall \ i,
\]

\[
T_o^j - V_o^j = \mu^j_o \left( \sum_{i \in I} \lambda^i_o \hat{A}^{ij} \right) \quad \forall \ j,
\]

\[
\sum_{j \in I} V_o^j = \mu^{I+1}_o \sum_{i \in I} \lambda^i_o \hat{D}^i.
\]

The last equation is clearly a linear combination the two previous equations, so we have a redundancy in the system, but using a Cross-Fratar balancing algorithm this is not a problem, it’s only a matter of fixing one of the parameters e.g. \( \mu^{I+1}_o \). The elements of the balanced matrices are

\[
A^i_o \hat{A}^{ij} = \lambda^i_o \mu^j_o \hat{A}^{ij} \quad \forall \ i, j
\]

\[
D^i_o = \lambda^i_o \mu^{I+1}_o \hat{D}^i \quad \forall \ i.
\]

Through this procedure we have retrieved the systems

\[
\begin{bmatrix}
A_o & D_o \\
C_o &
\end{bmatrix}
\]

with all the linear relations required for the systems being fulfilled. These relations are:

\[
\sum_{i \in I} A^{ij}_o + \sum_{k \in K} C^{kj}_o = \sum_{i \in I} A^{ji}_o + D^j_o \quad \forall \ j.
\]

We now need data on labor usage by sector and region, and factor prices by factor and region. One has to keep in mind that the factor prices for labor and the labor usage has to add up to the labor costs of the firms. The labor costs of within each sector has already been obtained in the balancing of \( C_o \).

Letting labor be represented as factor type one i.e. \( k = 1 \), then \( C^{1j}_o \) is the total input cost of labor in sector \( j \). And it holds that:

\[
C^{1j}_S = \sum_{r \in S} L^{rj}_o \bar{w}^1, \quad \text{and} \quad C^{1j}_D = \sum_{r \in D} L^{rj}_o \bar{w}^1 \quad \forall \ j.
\]

We have made a simplifying assumption at this point. We have assumed that wages do not vary over all regions, but are the same within the subsets \( S \) and \( D \), i.e. we have only two factor prices for labor, one price for Swedish regions and another for the Danish. On the Swedish side, we have a set of regional
data, found in Örestat, on the number of people employed \(\{L_r\}_{r \in S}\) within the municipalities of Scania. We can find the Swedish factor price for labor through

\[
\bar{w}_S^1 = \frac{\sum_i C_{i}^{1j} S}{\sum_{r \in S} L_r}.
\]

Then the labor inputs per sector of Scania are given as

\[
L_S^i = C_{i}^{1i} S / \bar{w}_S^1, \quad \forall i.
\]

There is no other a priori information on labor usage for the Swedish part of the Öresund region. Again we use a matrix balancing approach, but this time the only information we have is information on row and column sums. We want to find a matrix \(L_r^i\) such that

\[
L_S^i = \sum_{r \in S} L_r^i, \quad \forall i,
\]

\[
L_r = \sum_{i} L_r^i, \quad r \in S,
\]

but in this case we don’t have any a priori matrix. Using balancing parameters \(\pi\) and \(\tau\), we want to find those satisfying

\[
C_{i}^{1i} S / \bar{w}_S^1 = L_S^i = \pi_S^i \sum_{r \in S} \tau_S^r, \quad \forall i,
\]

\[
L_r = \tau_S^r \sum_{i} \pi_S^i, \quad r \in S,
\]

and the Swedish part of the elements of the labor usage matrix are

\[
L_r^i = \pi_S^i \tau_S^r, \quad \forall i \text{ and } r \in S.
\]

For the Danish part of the labor usage matrix the same type of balancing is performed, but in this case we have an a priori matrix \(\tilde{L}\) from Örestat. Hence, we want to find the corresponding balancing parameters such that

\[
C_{i}^{1i} D / \bar{w}_D^1 = L_D^i = \pi_D^i \sum_{r \in D} \tau_D^r \tilde{L}_r^i, \quad \forall i,
\]

\[
L_r = \tau_D^r \sum_{i} \pi_D^i \tilde{L}_r^i, \quad r \in D,
\]

and

\[
L_r^i = \pi_D^i \tau_D^r \tilde{L}_r^i, \quad \forall i \text{ and } r \in D.
\]
This rebalancing of the labor matrix for the Danish side is performed to ensure that the required relation

\[ \bar{w}_D^1 = \frac{\sum_j C_{D}^{1j}}{\sum_{r \in D} L_r} \]

holds. In this balancing process we have treated the transport sector as any other, this is done for calibration purposes. When using the data though, the data for the transport sector will also be aggregated into the service sector data.

From a business survey conducted within the larger study of the impacts of the Öresund bridge, we know that approximately 65% of firms, located within the Öresund region, purchases are bought from outside this region. Using this knowledge and assuming trade balance for all sectors with the rest of the world, we set exports by sector \( \bar{E}^i \) equal to imports \( \bar{S}^i \), and both equal to 65% of total sectoral intermediate purchases. Furthermore, we know that cross strait trade amount to approximately 5% of the Öresund firms total trade, this is the trade quota \( \bar{Q} \) that we will use to calibrate the barrier parameter.

### 3.2.2 Calibration

If the data for the modeled economy are provided for the economy as a whole, then the calibration equations stated by Bröcker can be used, i.e.

\[
A^{ij} = \sum_s q_s^i a_s^{ij} x_s^j, \quad D^i = \sum_s q_s^i d_s^i,
\]

\[
C^{kj} = \sum_s w_s^k c_s^{kj} x_s^j.
\]

In Section 3.2.1 we discussed the construction of two parallel sets of data. In this way we have the IO-data given on a regional level, in the sense that we have input tables for both the Swedish and the Danish parts of the Öresund region. Hence, we need to regionalize the calibration equations. We use the equations:

\[
A_{S}^{ij} = \sum_{s \in S} q_s^i a_s^{ij} x_s^j, \quad A_{D}^{ij} = \sum_{s \in D} q_s^i a_s^{ij} x_s^j, \quad (16)
\]

\[
D_{S}^i = \sum_{s \in S} q_s^i d_s^i, \quad D_{D}^i = \sum_{s \in D} q_s^i d_s^i, \quad (17)
\]

\[
C_{S}^{kj} = \sum_{s \in S} w_s^k c_s^{kj} x_s^j, \quad C_{D}^{kj} = \sum_{s \in D} w_s^k c_s^{kj} x_s^j. \quad (18)
\]
These equations state that the regional $A^{ij}_S$ is interpreted as the total value of purchases from sector $i$ of all regions, to be used for production in industry $j$ in country $S$. Hence, $A^{ij}_S$ and $D^{i}_S$ are thought to include all imports from the Danish to the Swedish part of the Øresund region, and clearly vice versa for $A^{ij}_D$. $C^{kj}_S$ is the total value of the use of input factor $k$ in sector $j$ in the Swedish side. In addition to this we calibrate against data on labor use by sector and region, and factor price data by factor type and region through

\[
\bar{w}_s^k = w_s^k, \quad (19)
\]

\[
L^i_s = c^i_s x^i_s. \quad (20)
\]

Export and import data are used by setting

\[
E^i = \sum_r p^i_r E^i_r \text{ and } S^i = S^i. \quad (21)
\]

The barrier parameter $\beta$ is calibrated using the trade quota

\[
\bar{Q} = \frac{\sum_{r \in S, s \in D} T^{rs} + \sum_{s \in S, r \in D} T^{rs}}{\sum_{rs} T^{rs} + \sum_{ir} p^i_r E^i_r + \sum_{is} s^i}, \quad (22)
\]

where we have defined

\[
T^{rs} = \sum_i p^i_r c^i_s (d^i_s + \sum_j a^{ij}_s x^j) \quad (23)
\]

which gives the value of all trade from region $r$ to $s$. The trade quota describes the fraction of cross strait trade to the total trade value in the model.

Calibration of the model is performed in order to set the level parameters

$\beta$ and $(\alpha^j_D, \alpha^j_S, \gamma^j_D, \gamma^j_S, \delta^j_D, \delta^j_S, \vartheta^i, \zeta^j) \ \forall j.$

Among the equilibrium and calibration equations there is a $(3I + 2)$ redundancy, we close the system by fixing parameter levels:

\[
\sum_r \vartheta^i_r = 1, \quad \sum_{r \in D} p^i_r x^i_r = \sum_{r \in D} x^i_r, \quad \sum_{r \in S} p^i_r x^i_r = \sum_{r \in S} x^i_r, \quad (24)
\]

\[
\sum_i \delta^i_S = 1, \text{ and } \sum_i \delta^i_D = 1. \quad (25)
\]

The simultaneous solution of the equilibrium equations (1)-(15) and the calibration equations (16)-(25) calibrates the model parameters to the data, and gives us the benchmark solution used for comparative statics.
4 Results

In this section, two scenarios relating to the Öresund bridge are analysed. The first scenario implements reduced transport costs across the Öresund strait, and the second scenario involves reductions of the trade barrier between the Swedish and the Danish part of the Öresund region. The model is calibrated to pre-bridge data from the year 1999, which gives the benchmark equilibrium. Comparative static results are given for the different scenarios and the welfare effects are measured by relative equivalent variations (REV). Relative equivalent variations are defined as the percentage increases of incomes in the benchmark equilibrium required by the households in order to achieve the same levels of utilities as experienced in the counterfactual equilibrium. If we interpret the unit cost, or unit expenditure function of the household as the consumer price index, then the changes in welfare are equivalent to the changes in real incomes of the households.

The transport cost reduction scenario is implemented by halving the transport costs between the regions of Malmö and Copenhagen. This amounts to reducing the distance between the two regions by 14 kilometers. All other cross strait distances were reduced by seven kilometers, the smaller reduction representing the presence of alternative routes. In Table 1 some regional results are displayed. All regions experience increases in wages as well as capital rents, which taken together constitute the positive effects on regional incomes as the factor endowments are fixed in supply and fully employed. Also, the regional price indices are all lowered, such that the real incomes are elevated even further. In particular the Malmö region experiences the largest impact from the transport cost reductions, due to the increased access to and from the economically large Copenhagen region. The income effects are noticeably larger for the Swedish side. This leads to convergence of wage levels between Swedish and Danish regions, since the Danish regions

<table>
<thead>
<tr>
<th>%</th>
<th>Income</th>
<th>Wage</th>
<th>Capital rent</th>
<th>Price Index</th>
<th>Real income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helsingborg (1)</td>
<td>0.16</td>
<td>0.15</td>
<td>0.17</td>
<td>-0.03</td>
<td>0.19</td>
</tr>
<tr>
<td>Kristianstad (2)</td>
<td>0.15</td>
<td>0.14</td>
<td>0.15</td>
<td>-0.03</td>
<td>0.18</td>
</tr>
<tr>
<td>Malmö (3)</td>
<td>0.39</td>
<td>0.37</td>
<td>0.41</td>
<td>-0.07</td>
<td>0.47</td>
</tr>
<tr>
<td>Copenhagen (4)</td>
<td>0.07</td>
<td>0.08</td>
<td>0.07</td>
<td>-0.02</td>
<td>0.09</td>
</tr>
<tr>
<td>West Zealand (5)</td>
<td>0.07</td>
<td>0.08</td>
<td>0.07</td>
<td>-0.02</td>
<td>0.09</td>
</tr>
</tbody>
</table>
experienced higher wages than the Swedish regions in the benchmark equilibrium. In Figure 7 the regional welfare effects are displayed. The regional

Figure 7: Welfare effects by region measured in relative equivalent variation (REV) as a consequence the transport cost reduction scenario.

results are close to the results found in Bröcker [5]. His calculated welfare effects, also achieved in a CGE framework, are: Kristianstad (0.38%), Malmö (0.5%), Copenhagen (0.08%), West Zealand/Storstrøm (0.06%).

Regarding the barrier reduction scenario, we recall that we have retrieved the barrier parameter $\beta$ from the calibration of the model. The barrier parameter is interpreted as a price wedge, or tariff equivalent, imposed on all trade across the Öresund strait. Calibration yielded $\beta = 1.17$, hence introducing a 17% price increase on trade between Swedish and Danish regions. The barrier level is consistent with the trade quota introduced in the calibration of the model. The trade quota is defined as the fraction of cross strait trade to the total trade in the model. In the benchmark equilibrium the trade quota is set to be five percent. In the barrier reduction scenario we reduce the price wedge by half, such that it incurs an price wedge of eight and a half percent on cross strait trade. Regional results are displayed in Table 2. First, we notice that the regional economic impacts of the barrier reduction are larger in magnitude than the impacts from transport cost reductions in the previous scenario. Also, the barrier reduction impacts are more symmetric within countries than in the previous scenario: the effects in the Swedish regions are similar to each other and the same holds for the Danish regions. The reason for this symmetry within each country is that the barrier price wedge does not bear any distance component. The differences in results between the two countries are mainly due to the differences in technologies, preferences, factor endowments, and factor prices. The smaller
Table 2: Regional results from barrier reduction, percent change.

<table>
<thead>
<tr>
<th>%</th>
<th>Income</th>
<th>Wage</th>
<th>Capital rent</th>
<th>Price Index</th>
<th>Real income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helsingborg (1)</td>
<td>5.70</td>
<td>5.34</td>
<td>5.97</td>
<td>-1.12</td>
<td>6.94</td>
</tr>
<tr>
<td>Kristianstad (2)</td>
<td>5.32</td>
<td>4.98</td>
<td>5.56</td>
<td>-1.05</td>
<td>6.48</td>
</tr>
<tr>
<td>Malmö (3)</td>
<td>5.90</td>
<td>5.52</td>
<td>6.18</td>
<td>-1.15</td>
<td>7.18</td>
</tr>
<tr>
<td>Copenhagen (4)</td>
<td>1.46</td>
<td>1.55</td>
<td>1.39</td>
<td>-0.40</td>
<td>1.90</td>
</tr>
<tr>
<td>West Zealand (5)</td>
<td>1.45</td>
<td>1.52</td>
<td>1.38</td>
<td>-0.37</td>
<td>1.85</td>
</tr>
</tbody>
</table>

differences for regions within each country are mainly due to the distribution of factor endowments. In Figure 8 the welfare effects in the barrier reduction scenario are displayed, showing the similarity of effects within each country, and the larger effects for the Swedish regions. In Table 3 the trade quotas are given for the benchmark equilibrium and the different scenarios. An additional scenario has been added, where the trade barrier is completely removed, i.e. the price wedge is set to zero percent. The latter is performed to investigate the full potential of removal of trade barriers within the model. In this case, trade will be governed by the Armington assumption, with equal regional preferences between regions, subject only to transport costs. The transport cost reduction has only small implications for the trade pattern between the Öresund regions. The 50% barrier reduction scenario on the other hand increases cross strait trade by 40%, and complete removal of the barrier almost doubles the cross strait trade in relative terms. The barrier price wedge can be viewed in terms of distance equivalents, i.e. the trans-

![Figure 8](image-url)  

Figure 8: Welfare effects as the barrier is reduced by 50%. The initial barrier amounts to a 17% tariff equivalent, which is halved.
Table 3: Trade quotas, trade between Swedish and Danish side of the Öresund region related to total trade of the region.

<table>
<thead>
<tr>
<th>%</th>
<th>Trade quota:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>5.0</td>
</tr>
<tr>
<td>Trade cost scenario</td>
<td>5.1</td>
</tr>
<tr>
<td>Barrier scenario (50%)</td>
<td>7.0</td>
</tr>
<tr>
<td>Barrier scenario (0%)</td>
<td>9.5</td>
</tr>
</tbody>
</table>

Port distance equivalent needed to raise after freight prices by 17%. With the given transportation friction rates the distance equivalent is approximately 500 kilometers, compared to the interregional distances in the model that ranges between 30 and 180 kilometers. The barrier reduction scenario involves a 250 kilometer cut in the distance equivalent, which in relation to the 7-15 kilometer distance reduction in the transport cost reduction scenario explains the order of magnitude of the difference between the results in the two scenarios. The welfare changes in the 50% barrier removal and the complete barrier removal scenario are shown in Figure 9.

![Graph of REV as barrier is reduced](image)

Figure 9: Regional welfare effects from barrier reductions.

In Table 4 the sectoral redistribution of output quantities of the firms are shown. The increase in primary factor prices, wages and capital rents, together with a decrease in prices of all pool goods in the Öresund area induce a shift toward production in the less primary factor dependent sectors. The service sector is the most demanding sector regarding primary inputs per output produced, and the sales & repair sector is the least primary inputs demanding sector. This relation holds for both the Swedish and the Danish
production technologies, and we see a clear shift away from the service sector and a shift toward sales & repair sector in quantity terms.
5 Conclusions

The most important result we have found in this study is the potential of barrier removal for cross strait trade. We have calibrated the cross strait barrier consistently within the modeled framework and found it to represent a 17% tariff equivalent. That is, cross strait trade patterns before the bridge appear to be subject to a 17% trade tariff in addition to transport costs. When reducing this barrier to half its initial value, cross strait trade increases by approximately 50%, from 5% of total trade in the model to 7%. Completely removing the barrier almost doubles cross strait trade. The welfare effects in the barrier scenario are positive for all regions, with symmetrical regional effects within each country, but with larger effects for the Swedish regions. The larger effects are a consequence of an increase in the accessibility to the economically larger Danish regions from the Swedish regions.

On the other hand, the results from lowering of transport costs show overall smaller economic effects than the barrier scenarios. The transport scenario effects on regional welfare are similar to those found by Bröcker [5]. In the transport cost scenario we find that all regions experience positive effects on incomes and lowering of regional price indexes affecting the regional real incomes positively. Also, we find that the Swedish regions experience larger effects than Danish regions do, and especially the Malmö region which is directly affected by the bridge.

The sales and repair sector experiences increases in sales quantities for both scenarios, and for both sides of the Öresund region. On the contrary, the effects on the service sector are negative. These results are due to the production technologies of the respective sectors and an increase in factor prices.
References


A SCGE Trade Barriers

A.1 The reduced SCGE model

In this section we discuss alternative ways of modeling trade barriers. The approach we have chosen is to reduce the size of the nonlinear system as far as possible, in order to close down on issues regarding barrier modeling within our framework. Three alternative representations are stated within the smaller framework.

Base model with barriers through technologies

In analogy with the model presented in Section 2 we now state the smallest framework within which we can study barrier specifications. The model consists of two regions, one is a rest of the world region of the same type as in the original model. The other region is an “ordinary” region which contains one firm, one transport agent, and one household. Hence, the model contains only one type of good being traded. The firm uses only one type of primary input (labor) and no other intermediate inputs. Furthermore, all transport costs has been set to zero. We will index the regions as 1 and $R$, where $R$ is the rest of the world region, and we drop the sector index.

The workings of the rest of the world region is similar to that in the full Öresund model where import supply prices are fixed

$$ p_R \equiv 1. \quad (26) $$

The export demand function has been simplified somewhat, it is also fixed

$$ E \equiv 0.5, \quad (27) $$

which may be seen in the original formulation as setting the elasticity $\varepsilon = 0$, and the level parameter $\zeta = 0.5$. The transport agent acts according to the same CES specification as the one in region 1, described below but without the barrier parameter and with zero transport costs.

$$ q_R = (\vartheta_1 + \vartheta_R)^{1-\frac{1}{\rho}}(\vartheta_1 p_1^\rho + \vartheta_R p_R^\rho)^{\frac{1}{\rho}} \Rightarrow \quad (28) $$

$$ t_{1R} = \vartheta_1 (\vartheta_1 + \vartheta_R)^{1-\frac{1}{\rho}}(\vartheta_1 p_1 + \vartheta_R p_R)^{\frac{1}{\rho}-1}. \quad (29) $$

The firm in region 1 uses only one primary input (one unit of labor) as input to production, the price of one unit of output is therefore equal to the wage of one unit of labor

$$ p_1 = w, \quad (30) $$

25
A.1 The reduced SCGE model

A SCGE TRADE BARRIERS

which is a result of the following assumptions:

\[ a_1 = 0, \quad c_1 = 1. \]

Hence, no intermediate goods are used in production and one unit of labor is required per unit of output.

The transport agent in region 1 acts according to a CES technology, now with zero transport costs,

\[ q_1 = (\beta \vartheta_1 + \vartheta_R)^{1-\frac{1}{\rho}} (\beta \vartheta_1 p_1 + \vartheta_R p_R)^{\frac{1}{\rho}}. \]  

(31)

Here, the trade barrier is modeled through shifts in the regional Armington preferences. The barrier parameter \( \beta \) influences the preferences, or rather the technology directly, as it enters into the position parameters (\( \vartheta \)) of the CES unit cost function. The fractional demand for region 1 output in region 1 is given by

\[ t_{11} = \beta \vartheta_1 (\beta \vartheta_1 + \vartheta_R)^{1-\frac{1}{\rho}} (\beta \vartheta_1 p_1 + \vartheta_R p_R)^{\frac{1}{\rho}} - 1. \]  

(32)

The household in region 1 owns an exogenously given endowment \( f \) of labor which is fully employed and it acts according to the expenditure function

\[ eh(q_1, u) = uq_1. \]

In equilibrium the total income (\( fw \)) is spent on consumption and the demanded quantity of good is given by

\[ d = \frac{fw}{q_1} = u, \]  

(33)

which simply states that the demanded quantity is equal to total income divided by the unit price of the good. The demanded quantity is also equal to the utility of the household in this simple version of the one good economy. Given the technology specified in this model the firm’s output equals the labor input by the factor market clearing condition (13), that is, \( f = x \). Using this and the market clearing condition for region 1 we get

\[ f = t_{11} d + t_{1R} E. \]  

(34)

Then we can solve the model numerically by inserting equations (26)-(33) into equation (34), and solving for the wage level \( w \) satisfying the model.

We now have a description of the base model that we will analyze. In this first statement of the model, we introduced barriers to trade through altering of the position parameters of the transport agent. Two alternative approaches will be described below.
Barriers as subsidy equivalents

The only part that needs to be changed compared to the model in the previous section is the way we treat the barrier. In the previous specification, the barrier to trade was modeled as a shift in the position parameters of the transport agent’s CES unit cost function. The idea behind that approach is that the barrier is thought of as a technological or preference obstacle to trade. The idea used in this section is that we can model trade barriers as subsidy equivalents. That is, we skew the trade patterns between regions through the introduction of a subsidy on output produced in the home region. It is not strictly a subsidy, since it does not need to be financed, hence we denote it as a subsidy equivalent. An increasing barrier is modeled through an increased subsidy on output of the home firm, so that region 1’s transport agent shifts toward output produced in region 1.

Since we are now using prices to skew trade patterns we can simplify the parametrization of the transport agents CES unit cost function. It can now be stated as

$$q_1 = (\vartheta_1 (p_1 / \beta)^\rho + \vartheta_R p_R^\rho)^{1/\rho}. \quad (35)$$

As the barrier parameter, or subsidy parameter $\beta$ is increased, the transport agent perceives the price of output produced in the home region is lowered and shifts his purchases accordingly. The trade coefficient now becomes

$$t_{11} = \frac{\vartheta_1}{\beta} \left( \frac{p_1}{\beta q_1} \right)^{\rho-1}. \quad (36)$$

The transport agent in region $R$ has been reformed in an analogous fashion, again without the barrier parameter. Replacing the corresponding four expressions in the base model described in the previous section gives us the model with the barrier modeled as a subsidy equivalent.

Barriers as tariff equivalents

This version of the model is very much related to the subsidy version. The difference is that we now treat the barrier as a tariff equivalent instead. The barrier is simulated through a tariff on imports to region 1. The higher the tariff, the higher the barrier, and the transport agent in region 1 shifts toward purchasing from the home region.

The CES unit cost function of the transport agent now gives

$$q_1 = (\vartheta_1 p_1^\rho + \vartheta_R (\beta p_R)^\rho)^{1/\rho}. \quad (37)$$
and the trade coefficient is

\[ t_{11} = \varrho_1 \left( \frac{p_1}{q_1} \right)^{\rho-1}. \]  

(38)

The transport agent in region \( R \) has been reformed in an analogous fashion, again without the barrier parameter. Replacing the corresponding four expressions for \( q_1, t_{11}, q_R \) and \( t_{1R} \) in the base model described earlier gives us the model with the barrier modeled as a tariff equivalent.

A.2 Results

The following results are obtained with the model parameters set as

\[ \varrho_1 = 0.1 \quad \text{Position parameter} \]
\[ \varrho_R = 0.9 \quad \text{Position parameter} \]
\[ \rho = -2 \quad \text{CES elasticity= 3} \]
\[ \varepsilon = 0 \quad \text{ROW elasticity} \]
\[ \zeta = 0.5 \quad \text{ROW demand level} \]
\[ f = 0.43 \quad \text{Households endowment} \]

The welfare measure we have used is the relative equivalent variation (REV), which for a specific region \( r \) is defined as

\[ REV_r \equiv 100 \left( \frac{eh(q_r, \hat{u}_r)}{eh(q_r, u_r)} - 1 \right). \]  

(39)

Here, \( \hat{u}_r \) denotes the counterfactual equilibrium level of utility, and all other variables are measured in the benchmark equilibrium. This measure can, in our model system, also be expressed as

\[ REV \equiv 100 \left( \frac{\hat{u}}{u} - 1 \right), \]  

(40)

where we have dropped the regional index since only region 1 holds a representative household. Hence, the welfare measure is simply the percentage change in utility. We will show results for the utility directly, as the barrier parameter is changed. Clearly, those results will hold qualitatively for the welfare measure too since this is a linear transformation choosing one of the equilibriums as benchmark. We have made model runs for a range of values of the barrier parameter

\[ \beta = 0.1, 0.35, \ldots, 20.1 \]
A.2 Results

Barrier through tariff equivalent

When the barrier is modeled as a tariff equivalent we find that utility is strictly decreasing as the barrier is increased. This result is in line with economic intuition, since the tariff is never actually collected and redistributed. One can think of the tariff as being wasted.

![Figure 10: Barrier as tariff equivalent. Increased barrier, decreased utility](image)

Barrier through subsidy equivalent

In the light of the previous results one would expect that modeling the barrier as a subsidy equivalent, an increased barrier would lead to increased welfare, and indeed it does.

![Figure 11: Barrier as subsidy equivalent. Increased barrier, increased utility](image)

In this model we have skewed the trade patterns through a fictitious subsidy. In analogy with the tariff, which is not actually collected, the subsidy
is not financed. This is one reason for the result. The subsidy is a gift from an almighty force, and the cheaper the household in region 1 can buy output from region 1 the more of its income can be used to by from the rest of the world at a fixed price.

**Barrier through technology**

The results in this section refers to the minimum base model with the barrier modeled through the technology parameters of the transport agent. In Figure 12 we can see that there is an optimal barrier in terms of utility.

![Utility vs barrier](image)

**Figure 12:** Utility versus barrier parameter, where barrier is modeled as parameter shift. There is an optimal barrier in terms of utility.

The technology of the transport agent represent the spatial preferences of the households, as they consume the pool good. Hence, changing this technology means that we change preferences. Therefore we can actually no longer make welfare comparisons for varying barrier parameters.

**Synthesis**

In terms of trade patterns all three models generate qualitatively the same results. As the barrier is increased the regions acts more as closed economies and trade between them tends to zero. In terms of trade patterns there seems to be no clear candidate of the models to be preferred before the others.

In terms of welfare or utility the different formulations of the barrier produce three completely different results. In the choice between the subsidy and the tariff formulations, the latter is preferred because of the economic interpretation: removing the barrier decreases the trade costs of overcoming the barrier.
Regarding the technology or preference approach we have already stated that measures of welfare may be distorted using this specification. Though, it is helpful to make the following interpretation. First, we recall that the transport agent in the tariff case acts according to

\[ ct(p_1, p_R) = (\vartheta_1 p_1^\rho + \vartheta_R (\beta p_R)^\rho)^{\frac{1}{\rho}}, \]

which can be rewritten as

\[ ct(p_1, p_R) = (\vartheta_1 p_1^\rho + \vartheta_R \beta^\rho (p_R)^\rho)^{\frac{1}{\rho}}. \]

What this shows is that we are free to choose whether to interpret the change of barrier \( \beta \) as a change in prices or as a change in the position parameters of the CES function. In this specific case, a tariff increase is analogous to a decrease in the parameter for the ROW price (\( \rho \) negative). Now, recall that the actions of the transport agents in the "technology" model is governed by eq. (31). The technology can be restated as

\[ ct(p_1, p_R) = \left( \frac{\beta \vartheta_1}{(\beta \vartheta_1 + \vartheta_R)^{1-\rho} p_1^\rho} + \frac{\vartheta_R}{(\beta \vartheta_1 + \vartheta_R)^{1-\rho} p_R^\rho} \right)^{\frac{1}{\rho}}. \]

As before, we can interpret an increase in \( \beta \) as a decrease in the position parameter for the ROW price, and hence as an analogue increase in the tariff. Regarding the first parameter in the CES-function, that is

\[ \frac{\beta \vartheta_1}{(\beta \vartheta_1 + \vartheta_R)^{1-\rho}}, \]

we note that it is not monotone in \( \beta \). In Figure 13 this parameter has been plotted for different values of \( \beta \).

In summary, we can interpret an initial increase in \( \beta \) as a subsidy on home products, and later on it becomes a tariff for the same product. We have already stated that we prefer the tariff approach to the subsidy approach. Since the technology approach in our case shows features of a model with both tariffs and subsidies, we may discard it for the same reasons as the subsidy model.

## A.3 Conclusions

Our conclusion is that it seems appropriate to model the barrier as a tariff. Of course one can use the technology approach as long as it does not include any subsidy elements, as was the case for us because of our specific parametrization of the transport cost function.
The argument for not choosing the subsidy equivalent barrier approach is primarily economic. We can interpret both the tariff and the subsidy in terms of costs of transport. In the tariff case, an increased barrier translates to an increased transport cost, while in the subsidy case an increased barrier translates into a cut in, or even negative, transport cost. The latter seems rather contradictory to the concept of a barrier.

The technology approach, which technically can be seen as a combination of the two other approaches and may suffer the same problems as the subsidy approach. Other problems of the technology approach are more conceptual. The transport technology describes the spatial preferences in the model. By changing preferences of the model we can in principle generate any welfare effects we want, as we can choose parametrization of the transport agents cost function freely.
B Notation

Indices and sets:

\[ i, j \in \mathcal{I} = \{1, \ldots, I\} \quad \text{Sector} \]
\[ r, s \in \mathcal{R} = \{1, \ldots, R\} \quad \text{Region} \]
\[ k \in \mathcal{K} = \{1, \ldots, K\} \quad \text{Factor} \]

The endogenous variables of the SCGE model are:

- \( x_r^i \) Output of sector \( i \) in region \( r \).
- \( d_j^s \) Demand of good \( j \) in region \( s \).
- \( p_r^i \) Unit-price of output from sector \( i \) in region \( r \).
- \( q_s^i \) Unit-price of pool good \( i \) in region \( s \).
- \( w_r^k \) Unit-price of factor \( k \) in region \( r \).
- \( a_{j}^{is} \) Input of pool goods of sector \( i \) per unit output of sector \( j \) in region \( s \).
- \( c_k^j \) Input of factor \( k \) per unit output of sector \( j \) in region \( s \).
- \( t_r^i \) Delivery of output of sector \( i \) in region \( r \) per unit pool good in region \( s \).
- \( y_s \) Household’s income in region \( s \).
- \( u_s \) Household’s utility in region \( s \).
- \( E^i \) Export demand for sector \( i \) by ROW.
- \( S^i \) Import supply for sector \( i \) by ROW.
<table>
<thead>
<tr>
<th>SCGE Öresund</th>
<th>Öreslat</th>
<th>IAPs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Manufacture</td>
<td>3. Manufacture of wood products, printing and pulp</td>
<td>4. Primary logging and related services activities</td>
</tr>
<tr>
<td>2. Manufacture</td>
<td>2. Manufacture of food, beverages and tobacco</td>
<td>1. Agriculture, hunting and related services activities</td>
</tr>
<tr>
<td>4. Sales and Repair</td>
<td>9. Sale and repair of motor vehicles etc.</td>
<td>36 Maintenance and repair of motor vehicles and motorcycles, retail sale of automotive fuel (50.2)</td>
</tr>
<tr>
<td>5. Services</td>
<td>10. Wholesale and commission trade, retail trade, exc. of vehicles</td>
<td>50 Maintenance and repair of motor vehicles and motorcycles, retail sale of automotive fuel (50.2)</td>
</tr>
<tr>
<td>6. Transport</td>
<td>13. Transport</td>
<td>60 Land transport, transport via pipelines (60.1-3)</td>
</tr>
</tbody>
</table>

**Table 5: Sectoral Aggregation Scheme**
Table 6: Regional Aggregation Scheme

<table>
<thead>
<tr>
<th>SCGE Oresund</th>
<th>Orestat</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Swedish regions</strong></td>
<td></td>
</tr>
<tr>
<td>Index</td>
<td>Region</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>Helsingborg</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Kristianstad</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Malmö</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td><strong>Danish regions</strong></td>
<td></td>
</tr>
<tr>
<td>Index</td>
<td>Region</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>4</td>
<td>Copenhagen</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>West Zealand/Storstrøm</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>
Table 7: ACD Input matrix for Swedish part of Öresund region (mSEK).

<table>
<thead>
<tr>
<th>Sectors</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Final Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24947</td>
<td>4271</td>
<td>9193</td>
<td>19812</td>
<td>2309</td>
<td>35566</td>
</tr>
<tr>
<td>2</td>
<td>2328</td>
<td>14518</td>
<td>698</td>
<td>5502</td>
<td>1683</td>
<td>18779</td>
</tr>
<tr>
<td>3</td>
<td>3862</td>
<td>1912</td>
<td>1414</td>
<td>24672</td>
<td>5818</td>
<td>6334</td>
</tr>
<tr>
<td>4</td>
<td>28975</td>
<td>8202</td>
<td>12397</td>
<td>94116</td>
<td>4955</td>
<td>50714</td>
</tr>
<tr>
<td>5</td>
<td>5057</td>
<td>1528</td>
<td>702</td>
<td>18980</td>
<td>3939</td>
<td>3820</td>
</tr>
<tr>
<td>Labor</td>
<td>13272</td>
<td>4820</td>
<td>11657</td>
<td>13956</td>
<td>5344</td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td>17838</td>
<td>8256</td>
<td>7772</td>
<td>22320</td>
<td>9978</td>
<td></td>
</tr>
</tbody>
</table>

Table 8: ACD Input matrix for Danish part of Öresund region (mSEK).

<table>
<thead>
<tr>
<th>Sectors</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Final Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12016</td>
<td>5342</td>
<td>8550</td>
<td>34037</td>
<td>2672</td>
<td>67189</td>
</tr>
<tr>
<td>2</td>
<td>3640</td>
<td>22538</td>
<td>1630</td>
<td>21593</td>
<td>3812</td>
<td>60350</td>
</tr>
<tr>
<td>3</td>
<td>3006</td>
<td>3153</td>
<td>2402</td>
<td>46651</td>
<td>9559</td>
<td>18594</td>
</tr>
<tr>
<td>4</td>
<td>34137</td>
<td>24489</td>
<td>32442</td>
<td>272616</td>
<td>9781</td>
<td>215594</td>
</tr>
<tr>
<td>5</td>
<td>6094</td>
<td>4047</td>
<td>1684</td>
<td>69822</td>
<td>9787</td>
<td>14810</td>
</tr>
<tr>
<td>Labor</td>
<td>29060</td>
<td>21019</td>
<td>21994</td>
<td>68176</td>
<td>25677</td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td>41853</td>
<td>32975</td>
<td>14663</td>
<td>76163</td>
<td>44957</td>
<td></td>
</tr>
</tbody>
</table>

Table 9: Labor amount by sector and region (Full time equivalents).

<table>
<thead>
<tr>
<th>Region</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16147</td>
<td>5864</td>
<td>14182</td>
<td>16978</td>
<td>6501</td>
</tr>
<tr>
<td>2</td>
<td>10766</td>
<td>3910</td>
<td>9456</td>
<td>11320</td>
<td>4335</td>
</tr>
<tr>
<td>3</td>
<td>34361</td>
<td>12479</td>
<td>30180</td>
<td>36130</td>
<td>13836</td>
</tr>
<tr>
<td>4</td>
<td>72709</td>
<td>53821</td>
<td>53928</td>
<td>179104</td>
<td>70752</td>
</tr>
<tr>
<td>5</td>
<td>14529</td>
<td>9279</td>
<td>12099</td>
<td>25562</td>
<td>6329</td>
</tr>
</tbody>
</table>

Table 10: Factor prices for primary inputs by type and region (mSEK).

<table>
<thead>
<tr>
<th>Sector</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor</td>
<td>0.216609</td>
<td>0.216609</td>
<td>0.216609</td>
<td>0.333108</td>
<td>0.333108</td>
</tr>
<tr>
<td>Other</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 11: Interregional distances in the Öresund region (km).

<table>
<thead>
<tr>
<th>Region</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>91</td>
<td>55</td>
<td>60</td>
<td>101</td>
</tr>
<tr>
<td>2</td>
<td>91</td>
<td>10</td>
<td>88</td>
<td>108</td>
<td>183</td>
</tr>
<tr>
<td>3</td>
<td>55</td>
<td>88</td>
<td>10</td>
<td>30</td>
<td>96</td>
</tr>
<tr>
<td>4</td>
<td>60</td>
<td>108</td>
<td>30</td>
<td>10</td>
<td>75</td>
</tr>
<tr>
<td>5</td>
<td>101</td>
<td>183</td>
<td>96</td>
<td>75</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 12: Elasticities and transport rates

<table>
<thead>
<tr>
<th></th>
<th>$\alpha_{zz}$</th>
<th>$\alpha_{m}$</th>
<th>$\alpha_{x}$</th>
<th>$\delta$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacture I</td>
<td>0.9</td>
<td>7</td>
<td>5</td>
<td>3</td>
<td>0.0002</td>
</tr>
<tr>
<td>Manufacture II</td>
<td>0.9</td>
<td>6</td>
<td>6</td>
<td>3</td>
<td>0.0002</td>
</tr>
<tr>
<td>Construction</td>
<td>0.9</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>0.0005</td>
</tr>
<tr>
<td>Sales &amp; Repair</td>
<td>1.3</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>0.0005</td>
</tr>
<tr>
<td>Services</td>
<td>2.0</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>0.0006</td>
</tr>
</tbody>
</table>
Dynamic Spatial CGE Frameworks
- Specifications and simulations

Marcus Sundberg

October 2005
Div. of Transport and Location Analysis,
Dept. of Transport and Economics,
Royal Institute of Technology,
SE-100 44 Stockholm, Sweden,
Abstract

In the present paper we consider different specifications of a dynamic spatial computable general equilibrium model. In particular we are interested in the effects of different, commonly used assumptions on transition dynamics of such a model. We have specified models with different assumptions regarding capital mobility, the utility specification of the households and assumptions regarding perfect foresight or myopia. A number of simulations have been performed with these models in order to observe policy responses and in order to be able to make specification comparisons of the different models based on those responses. We consider the time it takes for the dynamic models to converge towards the long term steady state to be of importance for welfare assessment of policies and find that convergence may be slow depending on both the model and the policy considered.
Contents

1 Introduction .................................................. 1
   1.1 Dynamics and specifications .............................. 1
   1.2 Forward looking and backward dependence .............. 1

2 The Model .................................................... 2
   2.1 Temporal decisions ...................................... 3
      2.1.1 The representative households .................. 3
      2.1.2 The aggregate households ....................... 5
      2.1.3 The myopic households ............................ 5
   2.2 Statics .................................................... 7
      2.2.1 The firms ............................................ 7
      2.2.2 Transports .......................................... 9
      2.2.3 Government ......................................... 9
      2.2.4 Markets ............................................. 10
   2.3 Numerical solution ...................................... 11
      2.3.1 Non-recursive ...................................... 11
      2.3.2 Recursive ......................................... 12
   2.4 Specifications of the test model ....................... 13

3 Results and Comparisons .................................... 16
   3.1 Non-recursive simulations ............................... 16
      3.1.1 Taxation .......................................... 17
      3.1.2 Transport cost reduction .......................... 19
      3.1.3 Population growth ................................ 22
      3.1.4 Specification comparisons ......................... 25
      3.1.5 Numerical errors ................................... 29
   3.2 Recursive simulations .................................. 31

4 Summary and Conclusions ................................... 33

A List of variables ........................................... 36
1 Introduction

1.1 Dynamics and specifications

The questions we want to shed some light on in the present paper are mainly twofold:

- Does dynamics matter?
- How does model specification affect dynamics?

The first question refers to two strands of economic general equilibrium modeling, static and dynamic modeling. Static modeling allows greater detailing, such as sectoral and regional disaggregation, than dynamic models do. This is simply due to computational issues. If dynamic transitions toward an economic steady state are rapid, then static modeling may suffice. If such transitions are slow dynamics may have something important to say.

Second, different assumptions and model specifications are often used in economic modeling. We are interested in the consequences of such assumptions. Does model specification matter, and if so how does different specifications relate to each other? Specifically we test commonly used assumptions regarding capital mobility and consumer behavior.

1.2 Forward looking and backward dependence

Dynamic economic models can largely be divided into models assuming myopic behavior and models incorporating forward looking behavior.

Forward looking behavior is often coupled with perfect foresight, that is, the economic actors have full information regarding all future states of the economy. In forward looking models, every economic event is related to two distinct points in time when adjustment in behavior occur. Let us think of an economic event as an economic policy. Then announcement of the policy and implementation of the policy constitute the two distinct points in time. Yet, this distinction is rarely made and announcement and implementation are assumed to be simultaneous. As illustrated in figure 1, announcement will change the behavior of the forward looking agents, who will adjust their behavior due to the economic implications of the announced policy. In deterministic models with forward looking dynamics announcement means commitment. That is, if a government announces a policy the economic agents in the model will act as if the government will commit to the announced policy. Implementation on the other hand, affects the structure of the economy. Changes in taxes or new infrastructure would be examples
2 The Model

With its origin in the Ramsey-Cass-Koopmans (\cite{5},\cite{13} and \cite{17}) tradition of optimal savings modeling, the model presented here is a straight-forward extension in terms of the number of regions and the number of economic actors in each region. Operational multi-regional and multi-sectoral models have largely been static type CGE models. The objective here is to take a static multi-regional multi-sectoral framework, such as the spatial CGE model by Bröcker \cite{3} and to incorporate dynamics, both forward looking and recursive, in the simplest possible fashion. A similar model has been presented by Devarajan and Go \cite{6}, later extended and applied by Diao and Somwaru \cite{7}. Their models incorporate optimal temporal decisions both by the firms and by the households. Households make consumption and savings decisions, and
firms make investment decisions intertemporally. Our model differs in the
treatment of investments. We have used the assumptions that the savings of
the households equal the investments and that the households owns the cap-
ital stocks. Hence, only the households make intertemporal decisions in our
model. Firms act as static profit maximizers under perfect competition using
capital, labor and intermediate inputs for their production. The use of inter-
mediate goods in production is incorporated in the static model by Bröcker
but not in the previously mentioned dynamic models, introducing interre-
gional business-to-business trade. Goods are transported between regions in
accordance with an Armington system of spatial preferences implying that
goods are demanded by origin of production.

The building blocks of the model are represented in the sections below as
well as schemes used for the numerical solution of the model. First we turn
to the intertemporal decision makers in the model.

2.1 Temporal decisions

Consumption versus investment decisions are made intertemporally using
three alternative specifications of the decisive agent. We refer to the differ-
ent specifications as the representative household approach, the aggregate
household approach and the myopic household approach.

2.1.1 The representative households

The representative households are modeled as forward looking agents, they
are forward looking in the dimensions of perfect foresight, and myopic in all
other dimensions. Myopic in this context means that households act with
static expectations in those dimensions. Examples where the households are
myopic in this sense would be with regard to different policies, the house-
hold expect policies to remain unchanged as long as no change has been
announced. Future incomes and budgets on the other hand are known to the
household, under the announced policies, and allow the households to use this
information for their intertemporal decisions. We assume the representative
household in region \( r \) to represent the actions of \( l_{rt} \) identical households at
time \( t \), each individual household achieving a level of utility \( u \) from consum-
ing a fraction \( 1/l_{rt} \) of the total regional consumption \( c_{rt} \). For now we take \( l_{rt} \)
as exogenous to the model, not modeling the regional population dynamics.
The representative household in region \( r \) chooses how to distribute consump-
tion over time in order to maximize the intertemporal utility \( u \) with discount
factor \( \beta \). Income that is not spent on consumption today is invested into
the capital stock \( k_{r(t+1)} \) to be used in future production. The maximization
problem of the representative household in region $r$ can be stated as

\[
\max_{\{c_{rt}, k_{r(t+1)}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t l_{rt} u(c_{rt}/l_{rt}),
\]

subject to:

\[
k_{rt} w_{rt}^K + l_{rt} w_{rt}^L + g_{rt} = p_{rt}^C c_{rt} + p_{rt}^K (k_{r(t+1)} - (1 - \delta) k_{rt}) \quad \forall t,
\]

\[
\lim_{t \to \infty} \beta^t u'(c_{rt}/l_{rt}) \frac{p_{rt}^K}{p_{rt}^C} k_{r(t+1)} = 0,
\]

given $k_0$.

The left hand side of the time $t$ budget constraint above is the household’s income and the right hand side is its expenses. Households earn their incomes by providing the production factors, capital $k_{rt}$ and labor $l_{rt}$, to the firms. They earn capital rents $w_{rt}^K$ and wages $w_{rt}^L$, and in addition to this they receive transfers $g_{rt}$ from the government. The income is used either for buying a composite consumption good at price $p_{rt}^C$, or for investments in the capital stock at the price $p_{rt}^K$. Also, investments at time $t$ equal the change in the capital stock between time $t$ and time $t+1$, accounting for the depreciation rate of capital $\delta$. The second condition in the optimization problem of the representative household is the transversality condition, necessary for optimality see [15].

Let $\lambda_{rt}\beta^t$ be the Lagrangian multiplier related to the time $t$ budget constraint. The first order conditions for the household in region $r$ are

\[
c_{rt} : \quad u'(c_{rt}/l_{rt}) = \lambda_{rt} p_{rt}^C,
\]

\[
 k_{r(t+1)} : \quad \lambda_{rt} p_{rt}^K = \beta \lambda_{r(t+1)} (w_{r(t+1)}^K + (1 - \delta) p_{r(t+1)}^K),
\]

\[
 \lambda_{rt}\beta^t : \quad k_{rt} w_{rt}^K + l_{rt} w_{rt}^L + g_{rt} = p_{rt}^C c_{rt} + p_{rt}^K (k_{r(t+1)} - (1 - \delta) k_{rt}).
\]

Combining the upper two first order conditions gives us our version of the Euler equation. The consumption and investment streams can now be determined through the use of the Euler equations and the budget constraints

\[
\frac{u'(c_{rt}/l_{rt}) p_{rt}^K}{p_{rt}^C} = \frac{u'(c_{r(t+1)}/l_{r(t+1)})}{p_{r(t+1)}^C} \beta \left( w_{r(t+1)}^K + (1 - \delta) p_{r(t+1)}^K \right), \quad (1)
\]

\[
k_{rt} w_{rt}^K + l_{rt} w_{rt}^L + g_{rt} = p_{rt}^C c_{rt} + p_{rt}^K (k_{r(t+1)} - (1 - \delta) k_{rt}). \quad (2)
\]

The utility function we have chosen has the property of constant intertemporal elasticity of substitution. We have

\[
u(c_{rt}/l_{rt}) = \frac{(c_{rt}/l_{rt})^{1-\tau} - 1}{1 - \tau},
\]
satisfying the Inada conditions\(^1\) if \(\tau > 0\), and \(\tau\) is the inverse of the intertemporal elasticity of substitution.

### 2.1.2 The aggregate households

As an alternative to the representative household approach, an aggregate household formulation of the consumption decision has been formulated and tested. The aggregate household approach we have tested have the same form used in [7]. The aggregate household maximization problem is completely analogue to that of the representative household, but with a slightly different objective function

\[
\max_{\{c_{rt}, k_{r(t+1)}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_{rt}),
\]

where \(u\) is the same constant intertemporal elasticity of substitution utility function as in the representative household case. The formulation does not take into account the effects of population size in the welfare function, which is consistent with the representative household approach under either the assumption of constant regional populations, or the assumption of myopic behavior with regard to population size. The aggregate household form of the Euler equation, corresponding to equation (1), is

\[
\frac{u'(c_{rt})p_{rt}^K}{p_{rt}^C} = \frac{u'(c_{r(t+1)})}{p_{r(t+1)}^C} \beta \left( w_{r(t+1)}^K + (1 - \delta)p_{r(t+1)}^K \right).
\]

(3)

Compared to the representative household formulation, the transversality condition in the aggregate household formulation is also altered. By setting the population size \(l_{rt}\) to one in the former formulation of the transversality condition, we get the right form in the aggregate household case.

### 2.1.3 The myopic households

A recursive version of the model is implemented through the assumption that the households act under static expectations. That is, the households think that their own actions will not influence the equilibrium of the economy, and that regardless of what actions they decide to take, the prevailing equilibrium will persist. Hence, the households do not realize to the future economic impacts of their own or others behavior. Furthermore we assume that the households try to achieve their steady state level of consumption and capital.

\(^1\)The instant utility \(u\) satisfies the Inada conditions \(\lim_{x \to 0^+} u'(x) = \infty\) and \(\lim_{x \to \infty} u'(x) = 0\)
stock in the next time period. Clearly this does not imply that they will succeed in achieving this steady state since they act under static expectations, while in fact their actions will affect the equilibrium. In line with the household formulation under perfect foresight, we think of the households as

solving the following problem:

\[
\max_{c_{rt}, c_{r(t+1)}}, k_{r(t+1)}} l_{rt}u(c_{rt}/l_{rt}) + \frac{l_{rt}\beta}{1-\beta} u(\tilde{c}_{r(t+1)}/l_{rt}),
\]

subject to:

\[
k_{rt}w_{rt}^K + l_{rt}w_{rt}^L + g_{rt} = p_{rt}^C c_{rt} + p_{rt}^K (k_{r(t+1)} - (1-\delta)k_{rt}),
\]

\[
k_{r(t+1)}w_{rt}^K + l_{rt}w_{rt}^L + g_{rt} = p_{rt}^C \tilde{c}_{r(t+1)} + p_{rt}^K (\delta k_{r(t+1)}),
\]

given \( k_t \).

The households maximize the utility of today \( u(c_{rt}/l_{rt}) \) and the discounted stream of expected future steady state utility. Note that we use the notation \( \tilde{c}_{r(t+1)} \) for future expected consumption, this is done since expected consumption may differ from the actually realized consumption in the next period. The first budget constraint is the same as in the forward looking case. The second budget constraint reflects the households expectations of the future, it is evaluated under todays prices as households act under static expectations, and future investments equal the depreciation rate of the future capital stock \( k_{r(t+1)} \), which is the assumed steady state condition.

Let \( \lambda_{rt} \) and \( \tilde{\lambda}_{r(t+1)} \frac{\beta}{1-\beta} \) be the Lagrangian multipliers related to the budget constraints. The first order conditions for the household in region \( r \) are

\[
c_{rt} : \quad u'(c_{rt}/l_{rt}) = \lambda_{rt}p_{rt}^C,
\]

\[
\tilde{c}_{r(t+1)} : \quad u'(\tilde{c}_{r(t+1)}/l_{rt}) = \tilde{\lambda}_{r(t+1)}p_{rt}^C,
\]

\[
k_{r(t+1)} : \quad \lambda_{rt}p_{rt}^K = \frac{\beta}{1-\beta} \tilde{\lambda}_{r(t+1)}(w_{rt}^K + \delta p_{rt}^K),
\]

\[
\lambda_{rt} : \quad k_{rt}w_{rt}^K + l_{rt}w_{rt}^L + g_{rt} = p_{rt}^C c_{rt} + p_{rt}^K (k_{r(t+1)} - (1-\delta)k_{rt}),
\]

\[
\tilde{\lambda}_{r(t+1)} : \quad k_{r(t+1)}w_{rt}^K + l_{rt}w_{rt}^L + g_{rt} = p_{rt}^C \tilde{c}_{r(t+1)} + p_{rt}^K \delta k_{r(t+1)}.
\]

Combining the upper three first order conditions gives us our myopic version of the Euler equation. The consumption and investment streams can now be determined through the use of the myopic Euler equations and the budget constraints

\[
\frac{u'(c_{rt}/l_{rt})p_{rt}^K}{p_{rt}^C} = \frac{u'(\tilde{c}_{r(t+1)}/l_{rt})}{p_{rt}^C} \frac{\beta}{1-\beta} (w_{rt}^K - \delta p_{rt}^K), \tag{4}
\]

\[
k_{rt}w_{rt}^K + l_{rt}w_{rt}^L + g_{rt} = p_{rt}^C c_{rt} + p_{rt}^K (k_{r(t+1)} - (1-\delta)k_{rt}). \tag{5}
\]
We use the last first order condition to substitute out $\tilde{c}_r(t+1)$, and use the relationship that the investment at time $t$ is given as $\Delta_r := k_r(t+1) - (1 - \delta)k_{rt}$. Then the household problem is completely expressed in terms of time $t$ variables which is needed for our recursive formulation. In the recursive form of the model equations (1)-(2) are replaced by (4)-(5).

We have chosen to call the Euler equation myopic, which may sound contradictory. We think of the households as planning with an infinite time horizon which is clearly non-myopic. But, they act only under the information of the current equilibrium, they perceive the equilibrium myopically, or with static expectations. Also, the intention of the households to achieve steady state in the next period is myopic, this assumption could easily be relaxed by pushing the planned steady state assumption toward infinity in time and introducing a sequence of future planned consumption $\{\tilde{c}_r(t+1)\}_{t=0}^{\infty}$ with corresponding budget constraints.

There are many different ways to incorporate a recursive structure into the model, the presented one is only one possible way. We will in general expect the present recursive formulation to adjust faster toward the steady state than the model with rational expectations, simply because the households, by assumption, always strive to achieve the steady state in the next period.

\section{2.2 Statics}

All economic agents except the household act myopically, or statically, optimizing their behavior within every time step of the model. Firms choose the mix of inputs to production in order to maximize instantaneous profits, and all markets clear at all times. For simplicity we suppress the time indices for all variables in the remaining description of the model. In order to do this we recall the notation

$$\Delta_r := k_r(t+1) - (1 - \delta)k_{rt},$$

such that $\Delta_r$ denotes the investment in region $r$. The part of the model described in this section is similar to the static spatial computable general equilibrium model presented in [3].

\subsection{2.2.1 The firms}

We distinguish between different types of firms, belonging either to the production sectors, the consumer good sector or the investment good sector. Firms in the production sectors produce goods that are transported and used either as intermediate inputs to production, or to satisfy final demand and investment demand by the households. Both the consumption and the
investment good sector act as mergers, creating single composite goods, the consumption good being consumed to the wealth of the households and the investment good being used to augment the capital stock.

A firm $j$ in region $s$ belonging to the production sector is responsible for the production of good $j$ in $s$. The technology used in production is represented through an unit cost function $f^j$, describing the minimum cost of the required inputs for the production of one unit of output. Intermediate inputs are bought at prices $(q^1_s, \ldots, q^I_s)$ and primary inputs are bought at $(w^L_s, w^K_s)$. In addition to this, firms pay taxes $\tau^i$ and $\tau^K$ on intermediate and primary inputs, we introduce the notation

$$\bar{q}^i_s = (1 + \tau^1)q^1_s, \ldots, (1 + \tau^I)q^I_s), \quad (7)$$

$$\bar{w}^i_s = (1 + \tau^L)w^L_s, (1 + \tau^K)w^K_s) \quad (8)$$

Prices equal marginal cost due to perfect competition and free entry in all sectors

$$p^j_s = f^j(\bar{q}^i_s, \bar{w}^i_s). \quad (9)$$

Then, according to Shephard’s lemma [19], the cost minimizing input coefficients per produced unit of output are

$$a^{ij}_s = \frac{\partial f^j(\bar{q}^i_s, \bar{w}^i_s)}{\partial \bar{q}^i_s}, \quad b^{kj}_s = \frac{\partial f^j(\bar{q}^i_s, \bar{w}^i_s)}{\partial \bar{w}^k_s}. \quad (10)$$

The required input of type $i$ good per unit of produced type $j$ good in region $s$ is captured in $a^{ij}_s$, and the required amount of the primary input factors is given by $b^{kj}_s$, where $k$ denotes either capital or labor.

Both the consumption good and the investment good are simply composites of the production goods. Again, pricing to marginal cost implies

$$p^C_s = f^C(\hat{q}^C_s), \quad p^K_s = f^K(\tilde{q}^K_s), \quad (11)$$

and by Shephard’s lemma

$$d^i_s = \frac{\partial f^C(\hat{q}^C_s)}{\partial \hat{q}^i_s}, \quad y^i_s = \frac{\partial f^K(\tilde{q}^K_s)}{\partial \tilde{q}^i_s}, \quad (12)$$

where $d^i_s$ is the fraction of type $i$ good per unit of consumption, and $y^i_s$ is the amount of good $i$ needed per unit of the composite investment good. Here we have used the notation

$$\hat{q}^C_s = (1 + \tau^{CI})q^1_s, \ldots, (1 + \tau^{CI})q^I_s), \quad (13)$$

$$\tilde{q}^K_s = (1 + \tau^{KI})q^1_s, \ldots, (1 + \tau^{KI})q^I_s) \quad (14)$$
where \( \tau_{Ci} \) and \( \tau_{Ki} \) are the commodity specific tax paid in final demand sectors. Note the difference between \( \tau^K \) and \( \tau_{Ki} \), where \( \tau^K \) is a tax paid by production firms due to rental of capital from the households and \( \tau_{Ki} \) is the tax paid by the investment composition sector.

### 2.2.2 Transports

Spatial preferences are described in an Armington system where goods are demanded with respect to the origin of production. The transport agent of a region is responsible for transforming output of all regions into a composite commodity, a pool good. The pool good of kind \( i \) in region \( r \) is made up of output of kind \( i \) from all regions. Transport costs are included in the model through the iceberg principle, according to which goods are partly used up in the process of transportation.

The activity of a transport agent is specified through a NCES unit cost-function \( \tilde{h}^i(\tilde{v}_s^i) \), where \( \tilde{v}_s^i \) is the vector of prices facing the agent in region \( s \). Due to the iceberg principle these prices are given by

\[
v_{rs}^i = p_r^i \exp(\eta^i z_{rs}),
\]

where \( v_{rs}^i \) is the price of good of type \( i \) from region \( r \), perceived by the transport agent located in region \( s \). The parameter \( \eta^i \) is the commodity specific transport rate and \( z_{rs} \) is the distance between region \( r \) and \( s \). Define \( h_s^i(\tilde{p}) := \tilde{h}(\tilde{v}_s^i) \), then in equilibrium the price of a pool good equal the price of the composite commodity including transport costs

\[
q_s^i = h_s^i(\tilde{p}).
\]

The delivery of sector \( i \) from region \( r \) to \( s \) per unit pool good of region \( s \) is

\[
z_{rs}^i = \frac{\partial h_s^i(\tilde{p})}{\partial p_r^i}.
\]

Regarding the technologies of transport agent \( i \) in region \( s \), we assume that there is a common underlying technology for all regions. That is, goods from different regions are perceived the same way in all regions. Clearly the actual quantities demanded, for producing one unit of pool good, will vary by both region of origin and destination since the prices \( v_{rs}^i \) do vary by origin and destination.

### 2.2.3 Government

The government is modeled in the simplest fashion, running a balanced budget at all times. At every instant in time different taxes "\( \tau \)" are collected,
such as income, consumption, and industry specific taxes

\[ g_r = \sum_l \left\{ \sum_r \tau^L_l \omega^L_r + \tau^K k_r w^K_r + \sum_{ir} q^i_l (\tau^C_i c^i_d + \tau^K_i \Delta y_r^i) + \sum_{ijr} \tau^q a^i_j x^r_j \right\} \]

(17)

All taxes being collected and then directly redistributed as transfers to the regions, where the transfers are proportional to the population sizes of the regions.

### 2.2.4 Markets

Outputs are determined by the I/O equations

\[ x^i_r = \sum_s z^i_{rs} (d^s_i c^s + y^i_s \Delta s + \sum_j a^i_j x^j_s), \]

(18)

where output is bought by the households for consumption or investment, or by the firms to be used as intermediate inputs. The households are working in the region of residence and are assumed to be immobile between regions, labor market clears locally in each region

\[ l_r = \sum_i l^L_i x^i_r, \]

(19)

implying regional wage levels. The market for physical capital is modeled in two alternative ways. First, assuming complete mobility of capital between regions, the aggregate capital market clear

\[ \sum_r k_r = \sum_{ir} k^K_i x^i_r, \]

(20)

implying a common price on capital across regions. Second, assuming complete immobility where capital is only used in the region where it’s owned

\[ k_r = \sum_i k^K_i x^i_r, \]

(21)

allowing for regional differences in capital prices, is tested as an alternative formulation. Equations (1),(2) and (6)-(20) constitutes the base version of our dynamic spatial computable general equilibrium model, including the assumptions of the representative households and the common capital market.
2.3 Numerical solution

We have stated a simple form of forward looking dynamic formulation of a standard SCGE-model. The solution is non-recursive in structure since the equilibrium at any time is dependent on both the future states of the economy as well as the past. Hence, the model is solved for all times simultaneously.

There is a literature on solution methods alone for this type of problems. A variant of the Fair & Taylor algorithm [9] is used in the Monash model [8] when run in non-recursive mode. The parametric path approach by Judd [14] is another alternative approach. For the model formulated here we have chosen to implement a Newton based solution method. We have used a direct Newton method, which could be enhanced through parallelization and the use of Krylov methods, see [10]. A recursive formulation, or version, of the model is also solved.

2.3.1 Non-recursive

In the non-recursive forward looking formulation we make use of some simplifying assumptions. We approximate the infinite horizon problem with a finite time horizon $T$. We assume that the economy is in temporal equilibrium, or steady state at time $t = T$. In steady state, the Euler equation (1) and the budget constraint (2) are replaced by their corresponding steady state versions

$$\beta \left( \frac{w^K_r}{p_r^K} + (1 - \delta) \right) = 1, \quad (22)$$

$$k_r w^K_r + l_r w^L_r + g_r = p^c_r c_r + p^K_r \Delta_r. \quad (23)$$

Let $\bar{x}_t$ denote the the vector of all variables in the time $t$ equilibrium, $\bar{z}_t$ all exogenous variables and $f()$ the collection of equations that has to be satisfied in every time step. Then, the problem of finding the intertemporal equilibrium can be stated as

given $\bar{z}_t \ t = 0, 1, \ldots, T$
find $\bar{x}_t \ t = 0, 1, \ldots, T$ s.t.

$$\begin{pmatrix}
    f(\bar{x}_0, \bar{x}_1, \bar{z}_0) \\
    f(\bar{x}_1, \bar{x}_2, \bar{z}_1) \\
    \vdots \\
    f(\bar{x}_{T-1}, \bar{x}_T, \bar{z}_{T-1}) \\
    f(\bar{x}_T, \bar{x}_T, \bar{z}_T)
\end{pmatrix} = 0$$
where we have used the notation $\bar{f}(\bar{x}_T, \bar{x}_T, \bar{z}_T)$ to denote the steady state equations satisfied in the last time period. A Newton method has been used to solve the system of equations. The typical sparse structure of the Jacobian matrix used in our Newton method is displayed in Figure 2, in the case of 20 time steps. The sparse structure is a result of the fact that the equilibrium equations at time $t$ only concerns equilibrium variables from times $t$ and $t+1$.

We have used the assumption of steady state at time $t = T$, implying that the steady state equations are required to be fulfilled. This means that temporal differences of all variables are assumed to be zero. Another approach would be to set the solution at time $t = T + 1$ equal to the steady state solution. This could be implemented by replacing $\bar{f}(\bar{x}_T, \bar{x}_T, \bar{z}_T)$ with $\bar{f}(\bar{x}_T, \bar{x}_{ss}, \bar{z}_T)$ in our scheme, where $\bar{x}_{ss}$ denotes the steady state solution.

Finally, we note that the transversality condition in the households maximization problem has not yet been used in the equilibrium model. This condition is implied by the steady state assumption at time $t = T$, if all variables are bounded and non-zero in steady state and assuming that the discount factor $0 < \beta < 1$.

2.3.2 Recursive

The assumption of static expectations, used in the myopic household specification, decouples the intertemporal system of equations stated in the non-recursive model into a recursive sequence of intratemporal equilibriums. Using a similar notation as introduced in the previous section, recalling that

\[\text{Figure 2: Typical sparse structure of the Jacobian matrix.}\]
2.4 Specifications of the test model

The sequence $\bar{z}_t$ denotes all exogenous variables we now have the following recursive scheme to find the sequence $\{x_t\}_{t=0}^T$ of intratemporal equilibriums given state: $\{\bar{z}_t, \bar{k}_t\}$

find $\bar{x}_t$ s.t.

\[
\bar{f}_m(\bar{x}_t, \bar{z}_t, \bar{k}_t) = \bar{0}
\]

\[
\bar{k}_{t+1} = (1 - \delta)\bar{k}_t + \bar{\Delta}_t, \quad \bar{\Delta}_t \subset \bar{x}_t,
\]

next recursive state: $\{\bar{z}_{t+1}, \bar{k}_{t+1}\},$

where the recursion is initiated at a capital stock vector $\bar{k}_0$, and $\bar{\Delta}_t$ is the vector of regional investments. $\bar{f}_m()$ denotes the set of intratemporal equations in the myopic model, where the Euler equation (1) and budget constraint (2) have been replaced by their myopic counterparts (4) and (5). Starting off in period 0, the scheme steps forward in time, solving for the intratemporal equilibrium and adding the implied investments to the capital stocks which gives the capital stock in the next period. Each intratemporal equilibrium $\bar{x}_t$ is itself solved for by some numerical method applied to $\bar{f}_m = \bar{0}$, we have used a Newton method.

2.4 Specifications of the test model

The test model that has been implemented was kept small enough such that it was easy to alter and to test the different versions of the model with regard to market clearing conditions, final demand agent, forward looking or myopic modes. We have chosen a three-region two-sector model, allowing us to study sectoral and regional effects.

It remains to specify the unit cost functions of the firms and the transport agents. In the test model all unit cost functions are of the constant elasticity of substitution (CES) type. The unit cost CES function $f$ at prices $\bar{p}$ can be written as

\[
f(\bar{p}, \bar{\alpha}) = (\alpha_1 p_1^{1-\sigma} + \ldots + \alpha_N p_N^{1-\sigma})^{1/1-\sigma},
\]

where $\bar{\alpha}$ is the vector of position parameters, $N$ is the number of elements in both vectors and $\sigma$ is the elasticity of substitution of the actor. In Table 1 the position parameters and the elasticities of all used CES functions are displayed. All parameters are the same for all regions such that the benchmark steady state solution is geographically symmetric and there is no regional specialization. We have complete symmetry of regional Armington preferences, which means that in both sectors the corresponding transport
agent has symmetric regional preferences. The technology of the goods producing firms are slightly different in the two sectors, sector one being more dependent on capital and sector two on labor.

Table 1: Unit cost function specifications

<table>
<thead>
<tr>
<th></th>
<th>Position parameter vector</th>
<th>Elasticity of substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production sector 1</td>
<td>[0.25 0.25 0.1 0.4]</td>
<td>0.5</td>
</tr>
<tr>
<td>Production sector 2</td>
<td>[0.25 0.25 0.4 0.1]</td>
<td>0.5</td>
</tr>
<tr>
<td>Consumption sector</td>
<td>[0.5 0.5]</td>
<td>0.5</td>
</tr>
<tr>
<td>Capital sector</td>
<td>[0.5 0.5]</td>
<td>0.5</td>
</tr>
<tr>
<td>Transport agents</td>
<td>[0.33 0.33 0.33]</td>
<td>3</td>
</tr>
<tr>
<td>Household</td>
<td></td>
<td>0.5</td>
</tr>
</tbody>
</table>

The intertemporal utility functions of the households are of CES-type, and the intertemporal elasticity of substitution relates to the elasticity parameter as $1/\tau$. Beaudry and van Wincoop [2] estimate this parameter to be "significantly different from zero and probably close to one". We have set this parameter to 0.5. The other elasticities have been chosen according to Table 1. The discount factor $\beta$ is set to 0.92 and the capital depreciation rate is set to 0.08.

Furthermore we have to specify the transport rates and the geography. Transport rates describe the transport friction in the iceberg transport costs, or the rate at which the iceberg melts away. We set the transport rates for the two types of goods to be $\eta^1 = 0.2$ and $\eta^2 = 0.3$. The geography is completely symmetric, as shown in Figure 3, with equal distances of 3 units between regions and intraregional distances of 0.5 units. The intraregional distance implies positive transport costs for trade within a region.
Figure 3: The symmetrical geography of the test model. The arrows show all possible geographical trade flows.
3 Results and Comparisons

Let us first recollect the model structure, or the different variations of the model stated in the previous section. We have stated a dynamic spatial computable general equilibrium model, where we have allowed for different specifications regarding capital markets and consumption decisions. Capital markets are either assumed to be completely regional where the capital is owned within the region of use, admitting regional capital rentals, or a common capital market is assumed with perfect mobility of capital. The consumption decision is either modeled through the representative household formulation or through the aggregate household approach. Also, the temporal aspects of the model are allowed to vary through either the use of perfect foresight dynamics (non-recursive) or static expectations dynamics (recursive). The results are presented in order to arrive at comparative conclusions regarding the different specifications of the model and also in order to show some possible policy responses of the model. We have chosen to mainly focus on consumption paths, both regional aggregate consumption and regional per capita consumption paths, since they constitute a natural foundation for different possible welfare indexes.

3.1 Non-recursive simulations

The simulations presented in this section have been performed with the model run in non-recursive mode, i.e. we have assumed perfect foresight. As discussed in Section 1, this implies that any policy or change in exogenous variables is associated with two distinct points in time. The first is the time of announcement and the second is the time of implementation. In the next sub-sections some policy simulations are shown, using the common capital market and representative household specifications, symmetric geography and parameters according to Section 2.4.
3.1.1 Taxation

The specification of the government states that taxes may be collected from different sources, such as taxes on consumption goods, capital goods or intermediate inputs to the firms. All tax revenue is then redistributed to the population, where we have chosen to transfer back an equal amount to each individual. In the particular simulation performed here, we have assumed all taxes to be zero initially. Eventually a consumption tax of ten percent is imposed on sector 1 goods while all other taxes remain zero, see Figure 4.

![Consumption tax increase on sector 1 goods.](image)

The modeled economy is at steady state equilibrium prior to the tax change. At time step \( t = 5 \) the tax policy is announced and it is implemented at time step \( t = 14 \). The imposed tax is distortive, increasing the unit price of consumption good 1 relative to other prices.

Some results are displayed in Figure 5. The top left figure shows the consumption path, the top right displays the capital path, and the two lower figures shows the outputs produced by the two producing sectors. All regions experience the same paths since we have spatial symmetry in the model. In the consumption path we notice the effect of the announcement and the implementation of the policy. When the policy is announced, the households substitute consumption intertemporally in favor of consumption prior to the tax increase. The new long term steady state solution states a lower level of capital stock than the pre-policy equilibrium which admits the consumers to further increase the pre-implementation consumption at the expense of the decreasing capital stock. At the time of implementation, the households switch their behavior from consumption at expense of the capital stock to net accumulation of capital. Note that the household consumption converges
toward the long term steady state, the straight line in the consumption plot, and they are almost the same some twenty time steps after the policy implementation. In the lower plots the effects on the goods producing industries are shown. The time profile of capital shows large similarities with the output profile of sector 1, as indicated in the lower left plot. We recall that sector 1 is more dependent on capital input, according to the technology specifications, than sector 2 is. The long term increase on sector 2 output is due to the substitution away from sector 1 goods toward sector 2 goods in the consumption composite. Although the outputs of both sectors are decreasing in the announcement period, the consumption is increasing in this period. This is made possible through the reduced demand of the investment composite.
3.1 Non-recursive simulations

3.1.2 Transport cost reduction

Spatial computable general equilibrium models have been applied numerous times in economic assessments of infrastructure projects, see for example [4], [11], and [12]. Usually the infrastructure investment is represented by a reduction of transport costs. The models are usually of the static type, providing comparative static analysis based on the pre- and post-investment equilibria. Our intention here is to simulate the temporal implications of such an infrastructure policy.

![Figure 6: Distance between two regions reduced.](image)

As shown in Figure 6, the transport costs between the two lower regions are reduced, in this case through a distance reduction. The base model is spatially symmetric, but due to the infrastructure policy this spatial symmetry is destroyed. There is still spatial symmetry between the two lower regions, such that their policy responses will be the same. In the plots in Figure 7 the responses of the two lower regions follow the solid lines, while the responses of the upper region follows the dashed and dotted line. In the simulation we have reduced the interregional distance between the two lower regions by one percent.

In some policies, announcement itself can be viewed as a policy instrument as the timing of the announcement will affect the economic outcome. In the case of infrastructure investments the role of announcement as a policy instrument is limited. This is so because most infrastructure projects are planned and implemented during an extended period of time. A new bridge, railway link or road simply will not allow itself to be implemented in one day, neither is it plausible that the project would or could be kept secret from the people. In the transportation cost simulation we have set the announcement time to $t = 5$, the announcement period extends over four time periods and the new infrastructure is fully implemented and ready to use at $t = 9$.

In Figure 7 the results concerning consumption, capital, and sectoral outputs are displayed. In the consumption plot the new long run post-policy equilibrium has been displayed through the straight lines in the plot. As can be seen consumption has far from adjusted to anything close to the new
3.1 Non-recursive simulations

Results referring to the directly affected lower regions are represented by solid lines and the results for the upper region are represented by dashed and dotted lines. When it comes to the consumption/investment decision, we see that the two lower regions, directly affected by the policy, increase consumption and decrease investment in the announcement period. The intuition behind this result is that these regions can use the post-policy reduced transport costs to rebuild the capital stock. Since we have used the common capital market approach in this simulation, the capital does not need to be used in the region where it is owned. The capital plot shows the location of the ownership.

The transported volumes of goods are displayed in Figure 8, where the results for sector 1 goods are shown on the left and sector 2 goods on the right. The plots show the amounts transported on the links between the regions, where the shortened link between the lower regions is illustrated by the solid
The transport volumes on the two other links are shown in dashed lines and are equal due to geographical symmetry. As expected the shortened link induces an increase in transports along this link since the transport agents substitute toward cheaper goods. It is interesting to note that the transported amounts converge towards their steady state levels almost instantaneously, in contrast to the results on consumption and outputs. The steady state level of transports has been calculated to be 1.230 and 1.217 for sector 1 goods, and 0.8354 and 0.8250 for the sector 2 goods. With respect to traditional results from static spatial general equilibrium models, the differences in convergence time noted above have some intriguing implications. A static model, or for that matter the steady state version of our model, will give the same results regarding pre- and post-policy transported amounts as the dynamic model. However, the welfare implications of the dynamic framework used here may differ significantly from the static case due to the slow adjustment toward the new steady state consumption levels.
3.1.3 Population growth

The regional populations are exogenously given in the model. Here we will examine the implications of changes in regional populations. Different types of scenarios could be investigated within the framework, such as nativity scenarios or regional migration scenarios. In regional migration scenarios one could take into account that people moving from one region to another would bring their part of the capital stock with them. We have studied a type of nativity scenario, where people enter and exit regions with zero capital. In the previous simulations population sizes have been symmetrically set to one for all regions. In this section we will simulate the effects of the population changes according to the profiles shown in Figure 9. This is the reason why we will study the effects on consumption both in aggregate regional terms and in regional per capita terms. We have arbitrarily set the announcement time of the population changes to \( t = 5 \), which is prior to any of the changes. The region represented by a solid line experience population changes at \( t = 8 \) and \( t = 10 \) and the region represented by a dashed line experience population growth at \( t = 9 \).

The population profiles affect the regional outputs in a distinct manner due to the full employment assumption as can be seen in Figure 10. Again sector 1 outputs show stronger similarities to the aggregate capital stock profile than does sector 2 because of the capital dependencies of the respective production technologies. The households in the regions experiencing population growth starts to save as soon as they become aware of the future change in order to assure future levels of consumption. The region that has constant population size manage to hold its consumption level quite constant during
3.1 Non-recursive simulations

Results and Comparisons

Figure 10: Population simulation results.

the simulated period. Yet, the long term implication is that the consumption level in this region will fall to the steady state level indicated in the consumption plot. Again we note that the transition toward the steady state solution is a time consuming activity similar to the case in the transport cost scenario but in contrast to that in the taxation scenario.

We now turn to the results concerning per capita consumption in the population simulation. We have extended the simulation period to 100 time steps from announcement in order to illustrate some interesting properties in the per capita consumption paths experienced in the different regions, see Figure 11. The relative smoothness of the per capita consumption paths, compared to the aggregate regional consumption paths, are due to the smoothing property of the concave individual instant utility function $u(c_{rt}/l_{rt})$ which incurs smoothing on the per capita level of consumption $c_{rt}/l_{rt}$. The per capita consumption plot shows a complete reversal of the ordering of regional per capita consumption after approximately 80 time steps. This means that the
region where people initially experience the lowest level of individual consumption will eventually experience the highest level, and vice versa. Again, as in the transport cost scenario, the slow transition of the economy toward its steady state may lead to other conclusions from welfare analysis in this dynamic setting compared to static models. Furthermore, the extended initial period of reordering of per capita consumption paths strengthens the idea that transition paths may be important in comparison to the steady state or static equilibrium.

Figure 11: Consumption per capita, transitions and steady state.
3.1.4 Specification comparisons

The previously described simulations were performed with one specific version of the model with a common capital market and representative households. In this section we make comparisons between model responses for different specifications regarding capital mobility and the consumer agent. In Section 2 we specified two alternatives for capital mobility and two alternatives for regional consumption. Capital is either taken to be completely mobile between regions generating a common capital market or it is assumed to be regionally bounded with regional capital markets allowing regional capital rents. The regional consumption decisions are modeled either through the representative household approach or through the aggregate household specification. Hence, we compare results from the four different combinations of model specifications given in Table 2. We will use the order of display of

<table>
<thead>
<tr>
<th>Common capital market</th>
<th>Regional capital market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Representative household</td>
<td>Representative household</td>
</tr>
<tr>
<td>Common capital market</td>
<td>Regional capital market</td>
</tr>
<tr>
<td>Aggregate household</td>
<td>Aggregate household</td>
</tr>
</tbody>
</table>

Table 2: Different model specifications

Table 2 when we show the different model results below. When comparing the plots in Figures 12 and 13, left and right comparisons will reveal effects of alternative assumptions concerning capital mobility, and upper and lower comparisons show effects of consumption specifications.

In all comparatory simulations we have used the same parameters, as specified in Section 2.4, and we have tested two of the previously described scenarios. First, we compare models subject to the population scenario and then we compare results for the transport cost scenario.

In Figure 12 the aggregate regional consumption paths are displayed for the four different models subject to the combined scenario. The long term steady states are also shown as straight lines. First we make some comparisons regarding specifications based on these plots. The upper plots refer to the representative household specification and lower plots to the aggregate household outcomes. The aggregate household paths exhibit smoother transition paths for aggregate consumption than the representative household paths. This is a consequence of consumption smoothing, where the aggregate household approach performs smoothing on the aggregate level of consumption while the representative household specifications strive to smooth per capita consumption. In the representative household models aggregate consumption shows greater resemblance with the population dynamics. In the
announcement period, the representative household models, the solid line region increase savings on behalf of consumption in order to satisfy the demand of the future growing population. The aggregate household approach admits increasing consumption in the same region, since future population sizes do not appear explicitly in its objective. Furthermore, the aggregate household approach and the representative household approach share the same long term steady state solutions. Mathematically, this fact can be seen through the steady state Euler equation (22) which is independent of the differences in instant utility formulations: both formulations satisfy the same steady state equations.

We now turn to the results concerning capital market specifications, where results for the the common capital market models are shown in the plots to the left and the regional capital market models are shown on the right hand side in Figure 12. Comparing these results we see that the transition

![Consumption vs time](image_url)

Figure 12: Consumption in the population scenario for the different specifications.
paths show strong resemblance, over the simulated period, between the two specifications. Yet, the long term steady states differ depending on capital mobility. The convergence time toward steady state also differs between the specifications, where the regional capital market models almost obtain their steady state levels within the simulated period, while the other models still have a long way to go. The initial transition dynamics are more robust to assumptions regarding capital mobility than the static results.

![Consumption per capita vs time](image1)

![Consumption per capita vs time](image2)

![Consumption per capita vs time](image3)

![Consumption per capita vs time](image4)

Figure 13: Consumption per capita in the population scenario for the different specifications.

The corresponding results in terms of consumption per capita can be seen in Figure 13. The results concerning consumption smoothing are reversed when looking at per capita consumption. The representative households are more effective in this respect, as expected. Some interesting results regarding per capita consumption appear when comparing models with different capital mobility. First we note that the simulated transition paths show strong resemblance, as was the case for aggregate consumption, when comparing
the left plots to the right. Secondly, we note that the results for regional per capita consumption in the long term steady state is completely reversed when comparing the specifications. The solid line region which reaches the highest per capita consumption level in the common capital market models will achieve the lowest steady state levels in the regional capital models. Again, initial transitions in the dynamic setting are more robust to capital mobility assumptions than the static results.

In the transport cost scenario we keep regional populations constant, hence we will not show results regarding the consumer agent assumptions. The representative household and aggregate household approach will give the same results when populations are constant and equal to one. In Figure 14 we show the results regarding the assumption of capital mobility. Again, the

![Graph showing consumption vs time](image)

**Figure 14:** Consumption in the transport cost scenario for the different capital mobility specifications.

same type of conclusions can be drawn regarding convergence times toward steady state as was done in the previous specification test under the population scenario. Transition paths are much alike during the simulated period, but in the common capital market approach convergence toward steady state takes a long time.

The purchases of the capital composite by the households induce transports, as the capital good is a composite of the transported pool goods. Here we have examined the effects of different assumptions regarding capital mobility. The capital mobility we study refers to the mobility of the capital stock held by households, not to the purchases of capital adding to the capital stock. One could state a model imposing transport costs on regional redistribution of the capital stock. Clearly this would complicate the model. One can view the performed exercises here as an extreme case of sensitivity
analysis of such an extension of the model, imposing zero or infinite costs on relocation of the capital stock used for regional production.

### 3.1.5 Numerical errors

Primarily we are interested in the approximation errors due to the truncation of the solution period in the non-recursive model formulation. We have approximated the solution of the infinite horizon problem with that of a finite horizon problem by imposing the steady state assumption at the finite time horizon $T$. For simplicity we have chosen to study the approximation error imposed on the regional consumption paths rather than on all equilibrium paths. A base run was made, covering 100 time steps, or 100 years. This base run will be considered as the exact solution and we denote the consumption paths of this run by $c_{rt}$. Then, model runs with shorter finite horizons ($T$) were performed and we denote the consumption paths of those approximations by $c_{rt}^T$. Pointwise errors are defined as

$$e_{rt}(T) := c_{rt}^T - c_{rt},$$

and we use the Euclidean norm ($l^2$) to study

$$||e(T)||_2 := \left( \sum_r \sum_{t=1}^{10} (c_{rt}^T - c_{rt})^2 \right)^{1/2}$$

for a range of finite horizons $T$. The simulation used here includes tax and infrastructure policies and population growth. All those exogenous disturbances are implemented in the first six time steps of the model, thereafter all exogenous variables are kept constant and announcement time is set to zero for all changes. In the simulation made here we have imposed both changes in policy variables and in the exogenous regional population sizes. Primary input taxes ($\tau^1, \ldots, \tau^I$) were increased from 0.3 to 0.35 in time period 5. Population sizes were changed from 1 for all regions to 1.05 for region one at time step 4 and to 1.02 for region two at time step 5, the population for region 1 was then dropped to 1.04 at time step 6. Interregional distances were changed from 3 units to 2.99 between regions one and two.

In Figure 15 we display the regional consumption paths on the left and the numerical errors on the right. In the plot of regional consumption paths, the solution $c_{rt}$ is displayed with solid lines and the approximate solution $c_{rt}^{10}$ is shown with dashed lines. The error of this solution is displayed in the right hand figure, where $||e(10)||_2$ corresponds to the leftmost point. We see that the error is reduced by a factor of ten as the solution interval is doubled.
The consumption paths of the regions

The $l^2$-error of the boundary approximation.

Figure 15: The figure on the left shows the regional consumption patterns for $t \in [1, 10]$. The right hand figure shows the numerical errors due to the finite boundary approximation.

The consumption paths of the regions

The $l^2$-error of the boundary approximation.

from $T = 10$ to $T = 20$ and then by another factor of ten as the horizon is redoubled to $T = 40$.

One could argue that the stated model is in fact a discretization of a time-continuous model. In that case it would be interesting to study the discretization error of the solution. Our model uses time steps of one year, but the solution may be sensitive to the size of time steps. In the article by Mercenier and Michel [16] this problem is studied for perfect foresight models. Among other things, they state conditions on model parameters such that the same underlying continuous problem is represented under different time discretizations. One can understand that a parameter such as the discount factor $\beta$ has to be appropriately chosen depending on the chosen step size in time. They use the concept of steady state invariance, to find such relations for parameters. This concept states that regardless of the time discretization chosen, the models should achieve the same steady state solution. We disregard this question and simply think of our model as a discrete model, not an approximation of a continuous one.
3.2 Recursive simulations

In this final section of simulation results, we will show the behavior of the recursive or myopic version of the model. Results regarding the three different scenarios (taxation, population and transport costs) covered in the forward looking simulations will be stated. All features of the previously used model are retained here, except that of myopia instead of perfect foresight. As the myopic behavior of households decouples their actions regarding investments from any future states of the economy, the concepts of announcement and announcement period play no role in this version of the model. Only implementations of policies or exogenous changes matter, and the adjustment of household behavior to the new economic circumstances is immediate, though this does not imply that the new steady state is achieved instantaneously as expectations are misguided.

In Figure 16 the regional consumption paths are shown for the taxation simulation and the transport cost scenario. In the taxation scenario consumption rises instantaneously as the consumption tax is increased. This is contrary to the result in the forward looking case with announcement (cf. Figure 5), where implementation incurs a sharp cut in consumption. Yet it is rational within the myopic setting, as the instant price signals to the consumers tell them to decrease the capital stock by using more of their income for consumption. In the tax scenario convergence toward the new steady state is achieved within the simulated period. In the transport cost reduction scenario the convergence toward the new steady state is slow, as

![Graph showing consumption vs time for recursive simulations.](image)
was the case in the non-recursive simulation (cf. Figure 7).

The population growth scenario also generates the same type of slow convergence behavior toward steady state as in the forward looking case, see Figure 17. In the right hand side plot we see that the initial reversion of regional per capita consumption levels compared to steady state levels is preserved in the myopic approach as well. Hence, slow convergence toward steady state seems to have little to do with announcement effects or assumptions regarding foresight. Rather, it seems that this convergence rate is dependent on the policy simulated, where the tax simulations, both in the recursive and non-recursive case, converges within the simulated period, while the other policy simulations require very long simulation periods to converge. Recall that these conclusions also depend on the assumption concerning capital mobility, as discussed in the previous section.

Figure 17: Consumption, and consumption per capita in the recursive population simulation.
4 Summary and Conclusions

We have stated and tested a number of different specifications of a dynamic spatial computable general equilibrium framework and performed policy simulations both in recursive and non-recursive versions of the model. Our implementation of the model has allowed us to separate the timing of policy announcement and policy implementation in the forward looking versions.

We have focused on studying regional consumption and regional per capita consumption paths, since we consider them fundamental to regional welfare analysis. Convergence times towards the steady state solutions are important when considering the performance of static models as compared to dynamic models in assessing welfare implications. The transition path may be more important than the steady state solutions if future welfare is to be discounted, and transition dynamics are slow. In the case of a common capital market we have observed that transitions towards the steady states are indeed slow and in the case of population changes the initial transitions even show a reversed regional consumption pattern compared to the steady states. The reversal of regional consumption patterns are upheld for about 80 years which may be considered as a long period of time. We have shown that initial transition paths are more robust to capital mobility than the long term steady states. Capital mobility reverses the regional ordering of per capita consumption in the long term steady state. Convergence times toward steady states are policy dependent, the taxation scenario shows fast convergence while the population and transport cost scenarios do not. In the transport cost scenario convergence of consumption to the steady state is slow, yet the transported quantities on the different links converge almost instantaneously. The results regarding the aggregate household or representative household approach are clear when considering dynamics versus statics. Static formulations are independent of the chosen approach, as they both satisfy the same set of equations. Only transition paths are affected. Numerical errors due to the finite horizon approximation has been specified and shown to be reduced by a factor of ten as the simulation period is doubled.
References


A List of variables

The endogenous variables of the models are:

Variables with expressed time indices
\[ c_{rt} \quad \text{Total consumption in region } r \text{ at time } t. \]
\[ k_{rt} \quad \text{Capital stock in region } r \text{ at time } t. \]
\[ p_{rt}^c \quad \text{Price of consumption composite in region } r \text{ at time } t. \]
\[ p_{rt}^K \quad \text{Price of investment composite in region } r \text{ at time } t. \]
\[ w_{rt}^K \quad \text{Capital rent in region } r \text{ at time } t. \]
\[ w_{rt}^L \quad \text{Labor wage in region } r \text{ at time } t. \]
\[ g_{rt} \quad \text{Government transfers to region } r \text{ at time } t. \]
\[ \Delta_{rt} \quad \text{Investments in region } r \text{ at time } t. \]

Variables with suppressed time indices
\[ q^i_s \quad \text{Unit-price of pool good } i \text{ in region } s. \]
\[ p^i_r \quad \text{Unit-price of output from sector } i \text{ in region } r. \]
\[ a_{ij}^s \quad \text{Input of pool goods of sector } i \text{ per unit output of sector } j \text{ in region } s. \]
\[ b_{sj}^k \quad \text{Input of factor } k \text{ per unit output of sector } j \text{ in region } s. \]
\[ d^j_s \quad \text{Demand of good } j \text{ in region } s \text{ per unit of consumption in region } s. \]
\[ y^j_s \quad \text{Demand of good } j \text{ in region } s \text{ per unit of investment in region } s. \]
\[ z_{rs}^i \quad \text{Delivery of output of sector } i \text{ in region } r \text{ per unit pool good in region } s. \]
\[ x^i_r \quad \text{Output of sector } i \text{ in region } r. \]